# ARITHMETIC:

# RULES AND REASONS.

BY

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CAMBRIDGE:
MACMILLAN AND CO.
LONDON: GEORGE BELL.
LIVERPOOL: DEIGHTON AND LAUGHTON.
MANCHESTER: R. BURGE.
1850.

# PREFACE.

THE teaching of the higher branches of Arithmetic forming part of my duties, as Mathematical Master of the Free Grammar School, Manchester, I could find no treatise which I considered suited to the wants of a school composed of boys intended for commercial pursuits, or for some profession; for though it was of importance to teach principles to all, the former required knowledge in certain commercial rules which it has of late been the custom to defer to a point which, although the place where they could best be discussed, was too far advanced for the boys who might be removed early from school, and never properly comprehended by others.

Whilst I admit the general principles on which they depend must always be there considered, still, at least in detail, all may be understood before; and, as the simple questions upon them would form better exercises than more numerous examples in the earlier rules, I deem the advantage in inserting them before Fractions will be easily perceived by those practically engaged in education. I have, however, endeavoured to make clear, when Fractions have been discussed, the general principles of all such cases, and have added more complicated questions, and other cases not to be understood before, or of much less importance. But I would recommend the study of the principles as soon as the intellect of the boy may be sufficiently matured. There can be no advantage

in the knowledge of many rules, or mere accuracy of calculation, when the reason of the process is unknown;—the understanding must be enlisted before the results obtained are acted upon in the affairs of daily life.

The merchant who, in the course of his business, performs the most minute calculations, I venture to say owes his knowledge far less to the rules he has learnt at school, than to his own mental powers having taught him the laws which apply to the cases he is ordinarily engaged in;—the subject is, in fact, too important to allow him to be the blind follower of a rule.

I have endeavoured to shew the reason of every rule I have given, but, as I conceive to mingle rules and reasons would only tend to confuse when the latter were beyond the power of the student, whilst I have given such explanations as I consider ought never to be omitted, I have kept the more difficult proofs distinctly separate, under every rule, but at the bottom of each page; the proof of the example preceding, in most cases, that of the rule.

The tutor will discover and direct the omission of proofs which, from the nature of the case, are too difficult of comprehension; and as I have, by my mode of printing, made this easily practicable, I may be excused in having given several entirely omitted in any Arithmetic which has come under my notice.

A collection of examples will probably follow; in the meantime I recommend "Thrower's," which appears, in most respects, well arranged.

*October* 15*th*, 1850.

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# TABLES.

## ADDITION.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |

## SUBTRACTION.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

## MULTIPLICATION AND DIVISION.

| 2 | | 3 | | 4 | | 5 | | 6 | | 7 | | 8 | | 9 | | 10 | | 11 | | 12 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 1 | 3 | 1 | 4 | 1 | 5 | 1 | 6 | 1 | 7 | 1 | 8 | 1 | 9 | 1 | 10 | 1 | 11 | 1 | 12 |
| 2 | 4 | 2 | 6 | 2 | 8 | 2 | 10 | 2 | 12 | 2 | 14 | 2 | 16 | 2 | 18 | 2 | 20 | 2 | 22 | 2 | 24 |
| 3 | 6 | 3 | 9 | 3 | 12 | 3 | 15 | 3 | 18 | 3 | 21 | 3 | 24 | 3 | 27 | 3 | 30 | 3 | 33 | 3 | 36 |
| 4 | 8 | 4 | 12 | 4 | 16 | 4 | 20 | 4 | 24 | 4 | 28 | 4 | 32 | 4 | 36 | 4 | 40 | 4 | 44 | 4 | 48 |
| 5 | 10 | 5 | 15 | 5 | 20 | 5 | 25 | 5 | 30 | 5 | 35 | 5 | 40 | 5 | 45 | 5 | 50 | 5 | 55 | 5 | 60 |
| 6 | 12 | 6 | 18 | 6 | 24 | 6 | 30 | 6 | 36 | 6 | 42 | 6 | 48 | 6 | 54 | 6 | 60 | 6 | 66 | 6 | 72 |
| 7 | 14 | 7 | 21 | 7 | 28 | 7 | 35 | 7 | 42 | 7 | 49 | 7 | 56 | 7 | 63 | 7 | 70 | 7 | 77 | 7 | 84 |
| 8 | 16 | 8 | 24 | 8 | 32 | 8 | 40 | 8 | 48 | 8 | 56 | 8 | 64 | 8 | 72 | 8 | 80 | 8 | 88 | 8 | 96 |
| 9 | 18 | 9 | 27 | 9 | 36 | 9 | 45 | 9 | 54 | 9 | 63 | 9 | 72 | 9 | 81 | 9 | 90 | 9 | 99 | 9 | 108 |
| 10 | 20 | 10 | 30 | 10 | 40 | 10 | 50 | 10 | 60 | 10 | 70 | 10 | 80 | 10 | 90 | 10 | 100 | 10 | 110 | 10 | 120 |
| 11 | 22 | 11 | 33 | 11 | 44 | 11 | 55 | 11 | 66 | 11 | 77 | 11 | 88 | 11 | 99 | 11 | 110 | 11 | 121 | 11 | 132 |
| 12 | 24 | 12 | 36 | 12 | 48 | 12 | 60 | 12 | 72 | 12 | 84 | 12 | 96 | 12 | 108 | 12 | 120 | 12 | 132 | 12 | 144 |

For Multiplication, say "twice one are two, twice two are four," &c.; "three times one are three," &c.

For Division, say "twos in two are one, twos in four are two," &c.; "threes in three are one," &c.

# MONEY TABLES.

### MONEY.

| | | | | | |
|---|---|---|---|---|---|
| | 2 | farthings(*f.* or *qr.*) | are | 1 | halfpenny. |
| | 2 | halfpennies | „ | 1 | penny (*d.*) |
| | 4 | pennies | „ | 4 | pence. |
| | 6 | pennies | „ | 6 | pence. |
| | 12 | pence | „ | 1 | shilling (*s.*) |
| 2*s.* 6*d.* or | 30 | pence | „ | 1 | half-crown.. |
| 5 0 „ | 60 | pence | „ | 1 | crown. |
| 10 0 | ... | ... | „ | 1 | half-sovereign. |
| 20 0 | ... | ... | „ | 1 | sovereign or pound (£). |

| FARTHINGS. | | | | | | PENCE. | | | | SHILLINGS. | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *far.* | | *half.* | | *d.* | | *d.* | | *s.* | *d.* | *s.* | | £ | *s.* |
| 2 | are | 1 | | | | 12 | are | 1 | 0 | 20 | are | 1 | 0 |
| 4 | or | 2 | are | 1 | | 18 | „ | 1 | 6 | 30 | „ | 1 | 10 |
| 6 | „ | 3 | „ | 1½ | | 20 | „ | 1 | 8 | 40 | „ | 2 | 0 |
| 8 | „ | 4 | „ | 2 | | 24 | „ | 2 | 0 | 50 | „ | 2 | 10 |
| 10 | „ | 5 | „ | 2½ | | 30 | „ | 2 | 6 | 60 | „ | 3 | 0 |
| 12 | „ | 6 | „ | 3 | | 36 | „ | 3 | 0 | 70 | „ | 3 | 10 |
| 14 | „ | 7 | „ | 3½ | | 40 | „ | 3 | 4 | 80 | „ | 4 | 0 |
| 16 | „ | 8 | „ | 4 | | 42 | „ | 3 | 6 | 90 | „ | 4 | 10 |
| 18 | „ | 9 | „ | 4½ | | 48 | „ | 4 | 0 | 100 | „ | 5 | 0 |
| 20 | „ | 10 | „ | 5 | | 50 | „ | 4 | 2 | 110 | „ | 5 | 10 |
| 22 | „ | 11 | „ | 5½ | | 54 | „ | 4 | 6 | 120 | „ | 6 | 0 |
| 24 | „ | 12 | „ | 6 | | 60 | „ | 5 | 0 | 130 | „ | 6 | 10 |
| 26 | „ | 13 | „ | 6½ | | 66 | „ | 5 | 6 | 140 | „ | 7 | 0 |
| 28 | „ | 14 | „ | 7 | | 70 | „ | 5 | 10 | 150 | „ | 7 | 10 |
| 30 | „ | 15 | „ | 7½ | | 72 | „ | 6 | 0 | 160 | „ | 8 | 0 |
| 32 | „ | 16 | „ | 8 | | 78 | „ | 6 | 6 | 170 | „ | 8 | 10 |
| 34 | „ | 17 | „ | 8½ | | 80 | „ | 6 | 8 | 180 | „ | 9 | 0 |
| 36 | „ | 18 | „ | 9 | | 84 | „ | 7 | 0 | 190 | „ | 9 | 10 |
| 38 | „ | 19 | „ | 9½ | | 90 | „ | 7 | 6 | 200 | „ | 10 | 0 |
| 40 | „ | 20 | „ | 10 | | 96 | „ | 8 | 0 | 210 | „ | 10 | 10 |
| 42 | „ | 21 | „ | 10½ | | 100 | „ | 8 | 4 | 220 | „ | 11 | 0 |
| 44 | „ | 22 | „ | 11 | | 102 | „ | 8 | 6 | 230 | „ | 11 | 10 |
| 46 | „ | 23 | „ | 11½ | | 108 | „ | 9 | 0 | 240 | „ | 12 | 0 |
| 48 | „ | 24 | „ | 12 | | 110 | „ | 9 | 2 | 250 | „ | 12 | 10 |
| 50 | „ | 25 | „ | 12½ | | 114 | „ | 9 | 6 | 260 | „ | 13 | 0 |
| 60 | „ | 30 | „ | 15 | | 120 | „ | 10 | 0 | 270 | „ | 13 | 10 |
| 70 | „ | 35 | „ | 17½ | | 126 | „ | 10 | 6 | 280 | „ | 14 | 0 |
| 72 | „ | 36 | „ | 18 | | 130 | „ | 10 | 10 | 290 | „ | 14 | 10 |
| 80 | „ | 40 | „ | 20 | | 132 | „ | 11 | 0 | 300 | „ | 15 | 0 |
| 90 | „ | 45 | „ | 22½ | | 138 | „ | 11 | 6 | 400 | „ | 20 | 0 |
| 96 | „ | 48 | „ | 24 | | 140 | „ | 11 | 8 | 500 | „ | 25 | 0 |
| 100 | „ | 50 | „ | 25 | | 144 | „ | 12 | 0 | 1000 | „ | 50 | 0 |

# WEIGHTS AND MEASURES.

### AVOIRDUPOIS WEIGHT.

| | |
|---|---|
| 16 drams (dr.) | 1 ounce (oz.) |
| 16 ounces | 1 pound (lb.) |
| 14 pounds | 1 stone (st.) |
| 2 stones or 28 pounds | 1 quarter (qr.) |
| 4 quarters or 112 pounds | 1 hundred weight (cwt.) |
| 20 hundred weight | 1 ton (t.) |

### APOTHECARIES' WEIGHT.

| | |
|---|---|
| 20 grains | 1 scruple (℈) |
| 3 scruples | 1 dram (ʒ) |
| 8 drams | 1 ounce (℥) |
| 12 ounces | 1 pound (℔) |

### TROY WEIGHT.

| | |
|---|---|
| 24 grains (gr.) | 1 pennyweight (dwt.) |
| 20 pennyweights | 1 ounce (oz.) |
| 12 ounces | 1 pound (lb.) |

### WOOL WEIGHT.

| | |
|---|---|
| 7 pounds | 1 clove. |
| 2 cloves (14lbs.) | 1 stone. |
| 2 stones (28lbs.) | 1 tod. |
| 6½ tods (128lbs.) | 1 wey. |
| 2 weys | 1 sack. |
| 12 sacks | 1 last. |

### LIQUID MEASURE.

| | |
|---|---|
| 4 gills or noggins | 1 pint. |
| 2 pints | 1 quart. |
| 4 quarts (8 pints) | 1 gallon. |
| 63 gallons | 1 hogshead. |
| 84 gallons | 1 puncheon |
| 2 hogsheads (126 gallons) | 1 pipe. |
| 2 pipes | 1 tun. |

### ALE AND BEER MEASURE.

| | |
|---|---|
| 2 pints make | 1 quart. |
| 4 quarts ,, | 1 gallon. |
| 36 gallons ,, | 1 barrel. |
| 54 gallons ,, | 1 hogshead. |
| 2 barrels (72 gals.) | 1 puncheon |
| 3 barrels (108 gals) | 1 butt. |

### CORN AND DRY MEASURE.

| | |
|---|---|
| 2 gallons (8 quarts) | 1 peck. |
| 4 pecks | 1 bushel. |
| 8 bushels | 1 quarter. |
| 5 quarters | 1 weigh or load. |
| 10 quarters | 1 last. |

### COAL MEASURE.

| | |
|---|---|
| 4 pecks make | 1 bushel. |
| 3 bushels ,, | 1 sack. |
| 36 bushels ,, | 1 chaldron. |
| 1 sack weighs | 224 lbs. |
| 10 sacks ,, | 1 ton. |

### CLOTH MEASURE.

| | |
|---|---|
| 2¼ inches | 1 nail. |
| 4 nails (9 inches) | 1 quarter. |
| 3 quarters | 1 Flemish ell. |
| 4 quarters | 1 yard. |
| 5 quarters | 1 English ell. |
| 6 quarters | 1 French ell. |

### LINEAL MEASURE.

| | |
|---|---|
| 3 barleycorns | 1 inch. |
| 4 inches | 1 hand. |
| 12 inches | 1 foot. |
| 3 feet | 1 yard. |
| 6 feet | 1 fathom. |
| 5½ yards (198 in.) | 1 rod (r), pole, or perch (p). |
| 40 poles (220 yds.) | 1 furlong. |
| 8 furlongs (1760 yds.) | 1 mile. |
| 3 miles | 1 league. |

¼ (one quarter) is used to shew that 4 must be taken to make up 1; ½ (one half) or two quarters, that 2 must be taken; ¾ means three quarters, and ¼, or one quarter more, would be 1.

MEASURE OF SURFACES.

| | | |
|---|---|---|
| 144 sq. inches | 1 | sq. foot. |
| 9 sq. feet | 1 | sq. yard. |
| 30¼ sq. yards | 1 | sq. rod, pole, or perch. |
| 40 perches | 1 | rood. |
| 4 roods (sq.yd. 1210) | 1 | acre. |
| 640 acres | 1 | sq. mile. |

MEASURE OF SOLIDS.

| | | |
|---|---|---|
| 1728 cubic inches | 1 | cubic foot. |
| 27 cubic feet | 1 | cubic yard. |

TIME.

| | | |
|---|---|---|
| 60 seconds | 1 | minute. |
| 60 minutes | 1 | hour. |
| 24 hours | 1 | day. |
| 7 days | 1 | week. |
| 4 weeks | 1 | lunar month |
| 12 calendar months | 1 | year. |
| 365 days | 1 | common year |
| 366 days | 1 | leap year. |

13 lunar months, or 52 weeks, are often considered a year.

## REMARKS ON THE TABLES.

*Money.*—The coins mentioned are all in circulation: the word guinea is often used instead of 21 shillings.

*Avoirdupois Weight* is used for all common goods.

*Troy Weight* is used for weighing gold, &c., and for experiments in philosophy.

*Apothecaries' Weight* is used in mixing drugs, which are sold by Avoirdupois.

*Liquid Measure.*—2 gills are considered half a pint in many places.

*Lineal Measure, &c.*—4 lineal perches or 100 links = 1 chain; 10 square chains or 100,000 square links = 1 acre.

*Time Table.*—The average length of the year being considered 365 days 6 hours, 3 common years and 1 leap year will come together; but as the true length is 365 days, 5 hours, 51 seconds, this would give too many leap years; and the correction is, when the century of the Christian Era cannot be divided by 4 without a remainder, it will be a common year.

By the *Calendar Months* are meant January, February, &c.—February has only 28 days.

Thirty days have September,
April, June, and November.

All other calendar months have 31 days. In the examples, when a lunar month is not intended, the length taken is 30 days.

# PRACTICE TABLES—ALIQUOT PARTS.

### OF £1.

| s. | d. | | |
|---|---|---|---|
| 10 | 0 | is | $\frac{1}{2}$ |
| 6 | 8 | " | $\frac{1}{3}$ |
| 5 | 0 | " | $\frac{1}{4}$ |
| 4 | 0 | " | $\frac{1}{5}$ |
| 3 | 4 | " | $\frac{1}{6}$ |
| 2 | 6 | " | $\frac{1}{8}$ |
| 2 | 0 | " | $\frac{1}{10}$ |
| 1 | 8 | " | $\frac{1}{12}$ |
| 1 | 4 | " | $\frac{1}{15}$ |
| 1 | 3 | " | $\frac{1}{16}$ |
| 1 | 0 | " | $\frac{1}{20}$ |
| 0 | 10 | " | $\frac{1}{24}$ |
| 0 | 6 | " | $\frac{1}{40}$ |

### OF 10 SHILLINGS.

| s. | d. | | |
|---|---|---|---|
| 5 | 0 | is | $\frac{1}{2}$ |
| 3 | 4 | " | $\frac{1}{3}$ |
| 2 | 6 | " | $\frac{1}{4}$ |
| 2 | 0 | " | $\frac{1}{5}$ |
| 1 | 8 | " | $\frac{1}{6}$ |
| 1 | 3 | " | $\frac{1}{8}$ |
| 1 | 0 | " | $\frac{1}{10}$ |
| 0 | 10 | " | $\frac{1}{12}$ |

### OF 6s. 8d.

| s | d. | | |
|---|---|---|---|
| 3 | 4 | is | $\frac{1}{2}$ |
| 1 | 8 | " | $\frac{1}{4}$ |
| 1 | 4 | " | $\frac{1}{5}$ |
| 0 | 10 | " | $\frac{1}{8}$ |
| 0 | 8 | " | $\frac{1}{10}$ |

### OF 5 SHILLINGS.

| s. | d. | | |
|---|---|---|---|
| 2 | 6 | is | $\frac{1}{2}$ |
| 1 | 8 | " | $\frac{1}{3}$ |
| 1 | 3 | " | $\frac{1}{4}$ |
| 1 | 0 | " | $\frac{1}{5}$ |
| 0 | 10 | " | $\frac{1}{6}$ |
| 0 | 6 | " | $\frac{1}{10}$ |
| 0 | 5 | " | $\frac{1}{12}$ |

### OF 4 SHILLINGS.

| s. | d. | | |
|---|---|---|---|
| 2 | 0 | is | $\frac{1}{2}$ |
| 1 | 4 | " | $\frac{1}{3}$ |
| 1 | 0 | " | $\frac{1}{4}$ |
| 0 | 8 | " | $\frac{1}{6}$ |
| 0 | 6 | " | $\frac{1}{8}$ |
| 0 | 4 | " | $\frac{1}{12}$ |

### OF 3s. 4d.

| s. | d. | | |
|---|---|---|---|
| 1 | 8 | is | $\frac{1}{2}$ |
| 0 | 10 | " | $\frac{1}{4}$ |
| 0 | 8 | " | $\frac{1}{5}$ |
| 0 | 5 | " | $\frac{1}{8}$ |
| 0 | 4 | " | $\frac{1}{10}$ |

### OF 2s. 6d.

| s. | d. | | |
|---|---|---|---|
| 1 | 3 | is | $\frac{1}{2}$ |
| 0 | 10 | " | $\frac{1}{3}$ |
| 0 | $7\frac{1}{2}$ | " | $\frac{1}{4}$ |
| 0 | 6 | " | $\frac{1}{5}$ |
| 0 | 5 | " | $\frac{1}{6}$ |
| 0 | 3 | " | $\frac{1}{10}$ |
| 0 | $2\frac{1}{2}$ | " | $\frac{1}{12}$ |

### OF 2 SHILLINGS.

| s | d. | | |
|---|---|---|---|
| 1 | 0 | is | $\frac{1}{2}$ |
| 0 | 8 | " | $\frac{1}{3}$ |
| 0 | 6 | " | $\frac{1}{4}$ |
| 0 | 4 | " | $\frac{1}{6}$ |
| 0 | 3 | " | $\frac{1}{8}$ |
| 0 | 2 | " | $\frac{1}{12}$ |

### OF 1s. 8d.

| | | | |
|---|---|---|---|
| 0 | 10d. | is | $\frac{1}{2}$ |
| 0 | 5 | " | $\frac{1}{4}$ |
| 0 | 4 | " | $\frac{1}{5}$ |
| 0 | 2 | " | $\frac{1}{10}$ |

### OF A SHILLING.

| d. | | |
|---|---|---|
| 6 | is | $\frac{1}{2}$ |
| 4 | " | $\frac{1}{3}$ |
| 3 | " | $\frac{1}{4}$ |
| 2 | " | $\frac{1}{6}$ |
| $1\frac{1}{2}$ | " | $\frac{1}{8}$ |
| 1 | " | $\frac{1}{12}$ |

### OF 10 PENCE.

| d. | | |
|---|---|---|
| 5 | is | $\frac{1}{2}$ |
| $2\frac{1}{2}$ | " | $\frac{1}{4}$ |
| 2 | " | $\frac{1}{5}$ |
| $1\frac{1}{4}$ | " | $\frac{1}{8}$ |
| 1 | " | $\frac{1}{10}$ |

### OF 6 PENCE.

| d. | | |
|---|---|---|
| 3 | is | $\frac{1}{2}$ |
| 2 | " | $\frac{1}{3}$ |
| $1\frac{1}{2}$ | " | $\frac{1}{4}$ |
| 1 | " | $\frac{1}{6}$ |

### OF 4 PENCE.

| d. | | |
|---|---|---|
| 2 | is | $\frac{1}{2}$ |
| 1 | " | $\frac{1}{4}$ |
| $\frac{1}{2}$ | " | $\frac{1}{8}$ |

### OF 3 PENCE.

| d. | | |
|---|---|---|
| $1\frac{1}{2}$ | is | $\frac{1}{2}$ |
| 1 | " | $\frac{1}{3}$ |
| $\frac{3}{4}$ | " | $\frac{1}{4}$ |
| $\frac{1}{2}$ | " | $\frac{1}{6}$ |
| $\frac{1}{4}$ | " | $\frac{1}{12}$ |

### OF A $\frac{1}{2}$ PENNY.

| d. | | |
|---|---|---|
| $\frac{1}{4}$ | is | $\frac{1}{2}$ |
| $\frac{1}{8}$ | " | $\frac{1}{4}$ |
| $\frac{1}{16}$ | " | $\frac{1}{8}$ |

### OF $\frac{1}{4}$ PENNY.

| d. | | |
|---|---|---|
| $\frac{1}{8}$ | is | $\frac{1}{2}$ |
| $\frac{1}{16}$ | " | $\frac{1}{4}$ |
| $\frac{1}{32}$ | " | $\frac{1}{8}$ |

### OF 1 CWT.

| | | |
|---|---|---|
| 2 qrs. | is | $\frac{1}{2}$ |
| 1 " | " | $\frac{1}{4}$ |
| 16 lbs. | " | $\frac{1}{7}$ |
| 14 " | " | $\frac{1}{8}$ |
| 7 " | " | $\frac{1}{16}$ |

### OF 1 QR.

| | | |
|---|---|---|
| 14 lbs. | is | $\frac{1}{2}$ |
| 7 " | " | $\frac{1}{4}$ |
| 4 " | " | $\frac{1}{7}$ |
| $3\frac{1}{2}$ " | " | $\frac{1}{8}$ |
| 2 " | " | $\frac{1}{14}$ |

### OF 1 LB.

| | | |
|---|---|---|
| 8 oz. | is | $\frac{1}{2}$ |
| 4 " | " | $\frac{1}{4}$ |
| 2 " | " | $\frac{1}{8}$ |

### LAND.—OF 1 ACRE.

| | | |
|---|---|---|
| 2 roods | is | $\frac{1}{2}$ acre |
| 1 " | " | $\frac{1}{4}$ " |
| 20 perches | " | $\frac{1}{8}$ " |
| 16 " | " | $\frac{1}{10}$ " |

### OF 1 ROOD.

| | | |
|---|---|---|
| 10 perches | is | $\frac{1}{4}$ rood |
| 8 " | " | $\frac{1}{5}$ " |
| 4 " | " | $\frac{1}{10}$ " |

## ERRATA.

Page 4. The sign for *therefore* should be $\therefore$, and not $\because$; also at p. 41, line 25; p. 46, lines 22 and 23; p. 47, line 6; p. 50, line 6; p. 53, lines 16, 22, and 31; p. 55, line 20; p. 64, line 14; p. 67, line 10.

„ 7, line 14, $5 \times 5 \times 5$, should be $5+5+5$.

„ 8, line 5, $3 \times 3 \times 3 \times 3 \times 3$, should be $3+3+3+3+3$.

„ 9, Ex. 3, line 4, should be 42408856; line 6, should be 24663778968.

„ 58, line 13, ·75000 should be ·7500.

„ 66, line 4, Ex. 2, 15000 should be 15·000, and ·625 should be ·0625.

„ 69, line 8, ·346 should be 3·46527; line 9, 573670 should be 57·3670.

„ 72, the parts of each rate should have been placed one line lower

„ 75, line 20, £3 17 4 should be £3 18 4.

„ 77, line 11, 2 w. should be 15 days.

„ 92, line 30, omit *there* before $^{1\frac{2}{5}0}$

# ARITHMETIC.

## NUMERATION.

1. By Arithmetic we are taught the use of numbers, and are instructed in finding the easiest and surest modes of calculating or reckoning.

2. To effect this ten digits or figures (called Arabic figures) are made use of, and rules are given so that we can represent by means of them any number whatsoever.

The first figure is written 1, and denotes the lowest number, and is called unit, unity, or one.

The next is written 2, and called two, and means one and one.

The remaining figures are 3 (three), 4 (four), 5 (five), 6 (six), 7 (seven), 8 (eight), 9 (nine), each increasing by one.

There is likewise another figure 0, denoting cipher, zero, or nothing.

3. To represent any number greater than 9 it is customary to write two or more of these figures together, and to make them increase in value tenfold as they stand further from the right hand.

This is called their local value.

Thus 10 does not mean 1 and 0, but the number next greater than 9, which is called ten—so 99 would me-

nine tens (or as it is called ninety) and nine, and is the largest number which can be expressed by two digits: the number next higher will be written 100.

Again, 100 means ten tens (or as it is called one hundred).

Similarly, 999 means nine hundred and ninety-nine: the number next higher will be 1,000 (one thousand).

4. Also these figures, as they stand in the first place from the right hand, are said to stand in the units place; figures in the second place, in the tens place; in the third place, in the hundreds place, &c., as is shewn in the upper part of the following table.

NUMERATION TABLE.

| Hundreds of Trillions | Tens of Trillions | Units of Trillions | Hundreds of Billions | Tens of Billions | Units of Billions | Hundreds of Millions | Tens of Millions | Units of Millions | Hundreds of Thousands | Tens of Thousands | Units of Thousands | Hundreds of Units | Tens of Units | Units |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 9 | 1 | 7 | 2 | 4 | 3 | 8 | 6 | 9 | 2 | 4 | 8 | 5 | 3 | 7 |
| | 5 | | | 4 | | | 3 | | | 2 | | | 1 | |
| | Trillions | | | Billions | | | Millions | | | Thousands | | | Units | |

The number being written 917,243,869,248,537, and called nine hundred and seventeen trillions, two hundred and forty-three billions, eight hundred and sixty-nine millions, two hundred and forty-eight thousands, five hundred and thirty-seven.

5. Numbers are generally divided into periods of three figures each, counting from the right hand, thus, 917243869248537 is written 917,243,869,248,537; this is shewn in the lower part of the table, and the names of the periods attached. The object of these periods is to make
ding of numbers easier.

There is another method by which numbers are divided into periods of six, which is not so convenient.

6. The signs and names of the numbers between 10 and 20 are the following:—

| | | | | | | |
|---|---|---|---|---|---|---|
| 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| eleven | twelve | thirteen | fourteen | fifteen | sixteen | seventeen |
| 18 | 19 | | | | | |
| eighteen | nineteen | | | | | |

The names and signs of the tens from 10 to 100 are

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 |
| twenty | thirty | forty | fifty | sixty | seventy | eighty | ninety |

7. Notation teaches us to write down any number in figures.

Numeration teaches us how to read these figures, and tell what number is represented.

8. Roman characters are often made use of to represent numbers; they are the following, and the numbers they express are written in the line above them:—

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| | I. | II. | III. | IV. | V. | VI. | VII. | VIII. | IX. |
| 10 | 11 | 12 | 13 | | | | | | |
| X. | XI. | XII. | XIII. | &c. | | | | | |
| | 20 | 30 | 40 | 50 | 60 | 70 | 80 | | |
| | XX. | XXX. | XL. | L. | LX. | LXX. | LXXX. | | |
| 90 | 100 | 500 | 1000 | | | | | | |
| XC. | C. | D. | M. | | | | | | |

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## QUESTIONS.

What is Arithmetic? What digits are used? What do they represent? Explain by how much each digit is greater than the one before? What is the meaning of 0?

How is the number next greater than 9 written? What is 99? What comes after 99? How much is one thouand? How much is a million? What is meant by units place? Tens place? Hundreds place? Write out the numeration table? How are numbers divided into periods? What is the use of periods? How are they counted? Give their names? What is notation? What is numeration? Write down the Roman characters as far as 10. What are the Roman characters for 34?

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## EXPLANATION OF SIGNS MADE USE OF.

$+$ Is read *Plus*, and shews Addition, or that the number following is to be added to that before.

$-$ *Minus*, the sign of Subtraction of the number which follows from that before.

$\times$ *Multiplied by*, the sign of Multiplication.

$\div$ *Divided by*, the sign of Division.

$=$ *Equal*, the sign of Equality between two numbers.

( ) Denotes that the number preceding or following has to be multiplied into every number contained between the lines.

$\therefore$ Is sometimes used for *therefore*; $\because$ for *because*.

# CHAPTER I.

## SIMPLE ADDITION.

9. When we have to find a number as great as two or more others taken together, we do it by addition.

Thus, "Add together 2 and 5," means what number is as great as 2 and 5 together, and the answer is 7.

RULE. Place the numbers under each other so that units stand below units, tens under tens, hundreds under hundreds, &c. Add the units together; set down the

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(9.) The numbers are placed units under units, &c., because we then add together figures having the same local value, and their sum will have the same local value; thus, if I have to add 53 and 84, I cannot add 8 to 3, because 8 here means eighty units, and 3 means only three units; but I can add 8 to 5 and get thirteen, if I remember that 8 means eighty, 5 means fifty, and 13, a hundred and thirty.

Thus, in the first example the figures in the units place represent six units and seven units, now these added together give thirteen units; the figures in the tens place represent ninety and and ten, these added together will give ten tens; the figures in the hundreds place represent seven hundred and four hundred, these added together give eleven hundred, similarly those in the thousands place will give twelve thousand. So the result of the addition is twelve thousand, eleven hundred, ten tens, and thirteen units. Now to write this down we must add that part which exceeds the units place to the tens, and similarly of the tens to the hundreds, and so on according to the words of the rule.

We thus get thirteen thousand two hundred and thirteen.

The same remarks apply to every example, and are a simple consequence of the law of notation, that the number in any place, when it exceeds nine, requires a higher place for its expression.

last figure of this sum in the units place of the answer; carry to add up with the figures in the tens place the remaining figure or figures, if any; place the last figure of of this second sum in the tens place; carry the others, if any, to add up with the hundreds, and continue in the same manner till all are added up.

| | | | | |
|---|---|---|---|---|
| Ex. 1. | 4716 | Ex. 2. | 3658 | PROOF. Add up, beginning at the top line and counting downwards, this must give the same answer. |
| | 8497 | | 8397 | |
| | 13213 | | 5940 | |
| | | | 7089 | |
| | | | 5268 | |
| | | | 30352 | |

## SIMPLE SUBTRACTION.

10. Subtraction teaches how one number may be taken away from one greater: thus, 2 subtracted from 6 means I must take 2 away from 6, which leaves 4.

RULE. Place the number to be subtracted below the other, with units under units, tens under tens, as in addition.

(10.) The reason of this rule is, that as every number consists of units, tens, hundreds, &c., in subtracting two numbers units must be taken from units, tens from tens, and so on, to get the answer; for the numbers which have the same local value may be subtracted from one another, as they have the same meaning; but if the units to be taken away be greater than the units of the other number, some more units must be added to the latter; we therefore borrow one from the tens place, that is ten units, but we must remember when we come to the tens that one ten has been made use of in the top line, and therefore the next figure in the top line must be diminished by one, or what is the same thing, the next figure in the bottom line increased by one.

The same reasoning exactly applies to the figures standing in higher places.

Subtract units from units, but if the number in the top line in the units place be less than the one below, ten must be borrowed and added to it, and then the difference be taken: proceed in the same way with the tens and hundreds, increasing the next lower figure by one, when we have been obliged to borrow just before.

| | |
|---|---|
| Ex. 13213<br>8497<br>——<br>4716 | PROOF. To prove the correctness of the working, add the remainder to the number subtracted; this will give the top line. |

## SIMPLE MULTIPLICATION.

11. By this rule we find how much a number will amount to taken (that is added to itself) any number of times: thus, 5 multiplied by 3 means 5 added to itself 3 times, or 5 × 5 × 5, that is, 15.

The number to be multiplied or taken the required number of times is called the multiplicand, and (the number of times it is to be taken, that is) the number by which it is to be multiplied, the multiplier, and the answer, the product: thus, 5 is the multiplicand, 3 the multiplier, and 15 the product.

(11.) Multiplication is merely a rule for performing addition when the numbers to be added are all alike. In Ex. 1, 7369 multiplied by 8 means 7369 added to itself eight times, and we apply our tables in the following manner: 7369 is 7000 + 300 + 60 + 9, and this added together eight times by our tables, or by actual addition (since 8 times seven is 56, &c.) is 56000 + 2400 + 480 + 72. Now to write this down we must carry on 70 to 480 which gives 550, and again 500 to 2400 which gives 2900, and 2000 to 5600 which gives 58000: we thus get 58000 + 900 50 + 2 or 58952, and we have done exactly as Rule 1 directs.

We may make the multiplicand the multiplier, and the product will be the same; but we must always take care to take as the multiplier the smaller number, as it makes the work easier: thus, 3 multiplied by 5 is the same as 5 × 3, as it is 3 × 3 × 3 × 3 × 3, or 15.

The multiplication table must be learnt to know how many the numbers below 13 will amount to, each repeated (or multiplied) as many times as possible below 13.

Rule 1. Place as in addition, multiply the units place in the top line by the multiplier (if less than 13), place the units down in the answer and carry the tens, next multiply the figure in the tens place and add the tens carried, write down the tens and carry the hundreds, and so proceed till all are multiplied.

| | |
|---|---|
| Ex. 1. | 7369 |
| | 8 |
| | 58952 |

Rule 2. If the multiplier be greater than 12, multiply by the figure in the units place, as in rule 1, then by the figure in the tens place, but place the first figure beneath the tens, and the next under the hundreds, &c.

If there be any more figures, the first figure of each fresh product must be put under that we are using as multiplier.

Add all the rows of figures up, as in addition, and the result will be the answer.

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In the next rule the reason the figures are moved one place higher is (taking Ex. 2 as a guide) that 3472 is not multiplied (or added together) 8 times, but 80 times; not 5, but 500; and the zeros, which would stand under the units, tens, &c. place, are left out in the process.

| Ex. 2. | | Ex. 3. |
|---|---|---|
| 3472 | | 6058408 |
| 587 | | 4071 |
| 24304 | | 6058408 |
| 27776 | | 44208856 |
| 17360 | | 242336320 |
| 2038064 | | 24653778968 |

It must be carefully remembered that where there is a cipher in the multiplier, it must be counted, and, as in example 3, 2 put under the third figure 8.

PROOF. Add the digits up in the multiplier and multiplicand, find the product; add all the digits together in the answer: divide each by nine. The remainders will be the same if the sum be worked correctly, if not, the sum is wrong (and may be wrong even when the remainders are the same).

## SIMPLE DIVISION.

12. In division one number which is the greater is to be divided by another, that is, the less is to be subtracted from the greater as many times as possible; and we are asked to find how often the less is contained in the greater, or how often the less number can be subtracted from it.

The greater number is called the dividend, the less the divisor, the answer the quotient, which tells how often the

(12.) Division is simply subtraction when the same number has to be subtracted as many times as possible, and we are required to find the number of times. To explain the rules, taking example Rule 1. for our guide; here we are required to subtract 8 from 73,683,509 as many times as possible. Now this is 73,000,000, 600,000, 80,000, 3,000, 500, and 9, but 8 will not subtract exactly from the parts excluding zeros of these numbers: it is better to put them under the form 72,000,000, 1,600,000, 80,000, 3,200, 240, 64, and 5, as we see at a glance that eight can be subtracted exactly

less is contained in the greater, or how often it may be subtracted from it.

The multiplication table must be made use of to find how often any number up to 12 is contained in any other number up to 144. Thus, if we are asked "*What is* 48 *divided by* 6?" find how often 8 is contained in 48; the tables tell us that 8 multiplied by 6, that is, taken 6 times, is 48, therefore 8 is contained 6 times in 48, or 48 divided by 8 gives 6.

SHORT DIVISION. RULE 1. When the divisor does not exceed twelve, write the divisor and dividend in the same line, with a mark or line between them. Find by the table how many times the divisor can be multiplied so as not to be greater than the first figures of the dividend. Put this number below, and carry on the number by which the first figures are greater than the multiplicand to the next figures, placing it before them: proceed as before, placing the next multiplier in the line below after the first: continue till all the figures have been divided, and place the last remainder, if any, with the divisor below it in a line with the quotient.

Ex. 1. 8)73683509

$9210438\frac{5}{8}$

Here 73 is found in the tables to give 9, and 1 remains to carry to 6 which makes 16, which we find in the tables to give 2 and no remainder: 8 gives 1 and no remainder: 3 we cannot divide, we therefore put zero and carry 3:

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from all but the last; but this is doing just as the rule tells us, namely, prefixing what the divisor will not exactly subtract (or be exactly multiplied into) in the first figures to the next figures, and so continuing. The result is 9,000,000, 200,000, 10,000, 400, 30, 8, with a remainder, 5, or 9,210,438 with remainder 5.

35 gives 4 and carry 3: 30 gives 3 and carry 6: 69 gives 8 and 5 remainder.

13. Long Division. Rule 2. When the divisor is more than 12, place the numbers as before with a line between them, and a line at the other side of the dividend. Find how many times the divisor may be multiplied by a number between 0 and 10, so as not to exceed the first figures of the dividend. Place the multiplier in the quotient, and write down the multiplicand under the first figures of the dividend, and subtract; bring down the next figure (or two if necessary) in the dividend, and place it after the remainder; find another multiplier as before, place it after the first figure in the quotient; subtract the multiplicand from the number in the bottom line, and proceed, in like manner, bringing down figures till they are all brought down, and write the remainder, if any, in the same line with the quotient, with the divisor under it. When two figures have been brought down at once, a zero must be written in the quotient before we write the next multiplier—when three, two zeros.

Ex. 2. Divide 9342 by 27

27 ) 9342 ( 346
81
——
124
108
——
162
162
——

Ex. 3. Divide 138716 by 34

34 ) 138716 ( $4079\frac{30}{34}$
136
——
271
238
——
336
306
——
30

(13.) Similar reasoning precisely applies to long division, the only difference being in the manner of writing down the process. The remainder is written, as directed, in the quotient, with the divisor under it, to shew that there remains something which we cannot divide any more.

Proof of both Rules. Multiply the quotient by the divisor and add the remainder, the result will be the dividend if worked correctly.

Proof for Multiplication. We may now prove multiplication, in all cases, by dividing the product by the multiplier; the quotient will be the multiplicand if the sum be right.

14. Rule 3. Contracted Division. When the divisor ends in one or more ciphers the following rule applies: —

Strike off the ciphers, and as many figures at the end of the dividend; divide the figures remaining as before; write the figures struck off at the right of the remainder, and place the number they form in the quotient with the divisor beneath it, including the zeros.

Ex. Divide 2625738 by 8100

81,00 ) 26257,38 ( $324\frac{1338}{8100}$
243
———
195
162
———
337
324
———
1338

This is simply an abbreviation for 8100 ) 2925738 ( $324\frac{1338}{8100}$
24300
———
19573
16200
———
33738
32400
———
1338

Divide 3567 by 20
20 ) 356,7
———
$178\frac{7}{20}$
———

15. Rule 4. When we can find two or more numbers less

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(15.) It is obvious that if we multiply the quotient by each of the divisors successively, that is, by the entire divisor, we shall get the dividend, which proves the correctness of the rule.

The remainders are written in the manner directed for the following reason:—The number written below the remainder denotes that the latter ought to be, but cannot be, divided any more: but the number in each line is the result of exact division; hence the remainders have undergone all former operations, but can be divided no more, and would therefore be written in the last line with the numbers by which each ought to have been divided below. But if we multiply by the divisors which have preceded (as in rule), then each ought to be divided by the entire divisor; and as then all the remainders ought to be divided by the entire divisor, we may add them together before shewing the division.

than 13, which multiplied together equal the divisor, the following rule is of use:—Divide by one of them as in short division, and the quotient by one of those remaining, and continue till all the numbers have been divisors. Write each remainder in a line with its quotient, with its divisor below. Multiply each remainder by all the divisors, except its own, which have preceded it. Add all the products together, and write their sum in the answer with the entire divisor below.

Divide 736498 by 48
Here $6 \times 8 = 48$

Hence $15343\frac{34}{48}$ is the answer.

$$\left.\begin{array}{r} 6\,)\,736498 \\ \hline 8\,)\,122749\frac{4}{6} \\ \hline 15343\frac{5}{8} \end{array}\right\}\frac{34}{48}$$

Here 5 (the last remainder) multiplied by $6 = 30$, then 4 must be added, which gives 34.

## QUESTIONS ON THE SIMPLE RULES.

What is Simple Addition? What does "add together 2 and 5" mean? What is the rule for addition? What is Subtraction? What does "subtract 2 from 6" mean? Give the rule for subtraction. What is Multiplication? What number is called the multiplier? What the multiplicand? What number the product? What does "5 multiplied by 3" mean? If I make the multiplicand the multiplier, will the answer be the same? Which of the numbers ought I to choose for the multiplier? What is the use of the multiplication table? Give the rules for multiplication. What is Division? Which number is called the divisor? Which the dividend? Which the quotient? Which the remainder? How must the multiplication table be used? In 48 divided by 6, what are we to find? Which is here the dividend? When must short division be used? What is the rule? How is the remainder to be written? When must long division be used? What is the rule? What is contracted division? What is the 4th rule in division?

# CHAPTER II.

## REDUCTION.

16. By Reduction we bring numbers denoting certain denominations of money, weights, or measures, to any required denomination.

**Rule** 1. To reduce a quantity of one denomination to another next lower. Multiply the former by the number

(16—18.) We have so far been concerned with mere numbers, or, as they are called, abstract numbers. But, as arithmetic teaches how all quantities may be represented by numbers, we are now about to consider how this may best be done.

The way is to call some quantity one, or write it as 1, then all quantities of the same nature will be represented by numbers,—thus, if we write one penny as 1, five pennies will be written 5, twenty pennies will be written 20, as five or twenty means five or twenty times one, but one is one penny.

But if we call one penny 1, it does not follow that twenty inches will be written 20, as, if it were so, we could have no means of telling whether 20 would mean money or inches, or some weight.

This difficulty is obviated by choosing a different unit for each thing of a different nature,—that is, we fix some quantity of money to be our one or unit, when money is concerned; another quantity to be our unit when length is concerned, as one inch; and another quantity for each thing of a different nature,—and either by words or some mark indicating whether a number means length, money, &c., when there is any likelihood of misapprehension.

Now, these units need not always be kept the same, for it is not always convenient, for instance, in money, to take one penny for the unit or one, for if we want to write 20 pounds, we cannot write it 20, but must consider how many pence we must write instead.

of things of the latter denomination there is in one of the former denomination. If a lower denomination still be required, reduce again to the denomination immediately lower, and continue till we arrive at the required denomination.

Ex. 1. Reduce £6 to shillings, also to pence.

$$\begin{array}{rl} \pounds\ 6 & \\ 20 & \\ \hline 120 & \text{shillings, because there are 20 shillings in a pound.} \\ 12 & \\ \hline 1440 & \text{pence, because there are 12 pence in a shilling.} \end{array}$$

17. RULE 2. To reduce a quantity of several denominations to one common denomination. Reduce the highest denomination to the one next lower; add the

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Reduction teaches how these units may be changed, and how we may write numbers meaning things of one kind (as pounds) as numbers meaning things of another kind, but of the same nature (as pence); thus, in the first example, £6 is required to be written as shillings; now, every pound contains 20 shillings, therefore £6 contains 6 times 20 shillings, or 120 shillings; hence, if I write £6 as shillings, I must multiply by 20, and write 120. This explains Rules 1 and 2.

Also, suppose I write 120, and it means 120 shillings, but I wish to consider one pound as my unit or one, 6 will then mean 6 pounds, or 120 shillings, and each one is now 20 times as great as it was, therefore I must divide by 20 to get a number which will mean the same. This explains Rule 3.

All such numbers are called concrete numbers.

It will be well to observe that when we are merely treating of abstract numbers, it may be sometimes convenient to change the unit; thus, supposing I have a number 150, if I agree that every ten of this number shall be represented by one, it may be written 15, for every unit or one is worth ten of the other units, therefore, 15 units are worth 15 times ten of the other units, or 150.

quantity in this denomination; reduce again, add as before, and proceed till all are reduced to the denomination required.

Ex. 2. Reduce £6 7*s.* 4*d.* to pence.

£ 6 7*s.* 4*d.*
20
———
127 shillings.
12
———
1528 pence.

18. Rule 3. To express a quantity of one denomination by another of higher denominations.

Divide by the number representing how many of this denomination there are in the next higher; the quotient will be in the next higher denomination, the remainder in the first denomination. Divide again by the number producing the next denomination, and proceed as far as required, recollecting that the quotient is always one denomination higher than the remainder,—the last quotient will represent the highest denomination required, and the remainders the successive lower denominations.

Ex. 3. Reduce 1528 pence to shillings and pounds.

12)1528*d.*
———
2,0)12,7*s.* 4*d.*
———
*Ans.* £6 7*s.* 4*d.*

It is often better to apply Rule 4 in division, thus, to bring 57302 ounces avoirdupois to pounds, &c.

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Again, suppose I did not choose to have so many as ten figures, but kept my unit just the same, and after, for instance, 6 instead of 7, I wrote 10 for 8, 11, and so on, it is clear I could represent all numbers so. This is called changing the scales of notation, and when there are ten figures made use of, numbers are said to be written in the scale of ten.

Here there are 16 oz., or 4 × 4 oz., in a lb.; 28 lbs., or 7 × 4 lbs., in a quarter; 4 qrs. in a cwt.; and 20 cwts. in a ton.

4) 57302

4) 14325...$\frac{2}{4}$ }
7) 3581...$\frac{1}{4}$ } 6 oz. Multiplying 1 by 4, and adding 2, we get 6, which will be oz.

4) 511...$\frac{4}{7}$ }
4)127...$\frac{3}{4}$ } 25 lbs. Here 3 × 7 is 21, adding 4, we get 25, which will be lbs.

2,0) 3,1...3

| | Cwts. | Qrs. | Lbs. | Oz. |
|---|---|---|---|---|
| Ton 1 | 11 | 3 | 25 | 6 |

In Lineal Measure, when we have to bring perches to yards, we must multiply by 11, and divide by 2; the remainder, if any, will be half a yard. In bringing yards to perches, we must multiply by 2, and divide by 11. The remainder, if any, will be in half yards.

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## COMPOUND ADDITION

19. Is the addition of like quantities, denoting money or some other article, but not reduced to the same denomination.

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(19—20.) These rules almost explain themselves: we place like denominations under each other, just as figures having the same local value, and, as they mean exactly the same thing, can add or subtract them as numbers; where, in adding, we get a number in one denomination amounting to more than are contained in a higher denomination, we carry, in compliance with custom, the excess to the next denomination: thus, in the example, the pounds (5 and 3) added, give 8 pounds, the shillings (9 and 16) give 25 shillings, the pence (6 and 8) give 14 pence, the farthings ($\frac{3}{4}$ and $\frac{1}{2}$) give 5*f*.; we do not write it £8 25*s*. 14*d*. 5*f*.; but since 25 shillings is £1 5*s*., 14 pence 1*s*. 2*d*., 5 farthings 1$\frac{1}{4}$*d*., we add one pound to the pounds, one shilling to the shillings, and one penny to the pence, and write £9 6*s*. 3$\frac{1}{4}$*d*.

RULE. Place quantities of like denominations under each other, the lowest being at the right hand. Add the quantities of the lowest denomination together; bring the result to a higher denomination; place the remainder under the last column; add the part in the higher denomination up with the next column, and continue till all the columns are added up.

Ex.

| £ | s. | d. |
|---|---|---|
| 5 | 16 | $6\frac{1}{2}$ |
| 3 | 9 | $8\frac{3}{4}$ |
| £9 | 6 | $3\frac{1}{4}$ |

PROOF.

| | Yds. | Ft. | In. |
|---|---|---|---|
| | 7 | 2 | $6\frac{1}{4}$ |
| | 6 | 1 | 8 |
| | 10 | 0 | $7\frac{3}{4}$ |
| | 5 | 2 | $3\frac{1}{2}$ |
| Yds. | 30 | 1 | $1\frac{1}{2}$ |

Add, beginning with the top row, and counting downwards.

## COMPOUND SUBTRACTION.

20. Here we have to subtract a quantity in several denominations from another of the same kind, but greater.

RULE. Place as in addition; subtract the quantity in the lowest denomination from the same quantity in the top line, adding in one of the next higher denomination

(20.) Similarly in the example in Subtraction, we cannot subtract $\frac{3}{4}$ from $\frac{1}{2}$, but from $1\frac{1}{2}$. But we must remember, when we subtract the pence, that we have borrowed one penny from the top line, and that there are really only five pennies left to be subtracted from. 8*d.* from 5*d.* (or 9*d.* from 6*d.*) is impossible; but borrowing a shilling, we then get the difference—9*d.*; we have, therefore, only 15 shillings left; we can subtract 9 from 15 (or 10 from 16), and we have 6*s.* left. We next subtract £3 from £5, and there remains £2; we have now subtracted the smaller quantity completely, and the result is £2 6*s.* $9\frac{3}{4}$*d.*

if necessary; continue, in like manner, adding one to the bottom line in the next operation, when one has been borrowed just before.

Ex. Subtract £3 9*s.* 8¾*d.* from £5 16*s.* 6¼*d.*

| £ | s. | d. |
|---|---|---|
| 5 | 16 | 6¼ |
| 3 | 9 | 8¾ |
| £2 | 6 | 9½ |

To prove, add the less to the answer,—the sum will be the greater.

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## COMPOUND MULTIPLICATION.

21. Here only one is a quantity representing money or any other article, and the multiplier is a mere number.

RULE 1. Place the multiplier under the number in the lowest denomination, which multiply, carrying as in compound addition, and so proceed with all the denominations.

RULE 2. When the multiplier consists of more than 12, we must try to find two numbers which, multiplied together, will equal it, and multiply by each in succession. If it cannot be so divided, the 1st rule must be applied.

A rule hereafter to be given (in Practice) will be the best when the number is very large.

Multiply £3 7*s.* 9¼*d.* by 6, and also by 18.

| £ | s. | d. | |
|---|---|---|---|
| 3 | 7 | 9¼ | |
| | | 6 | |
| 20 | 6 | 7½ | Ans. in the 1st case. |
| | | 3 | Because 6 × 3 is 18. |
| £60 | 19 | 10½ | Ans. in the 2nd case. |

PROOF.

Reduce the product to one common denomination, divide by the multiplier, and bring the quotient to higher denominations,—this ought to give the multiplicand.

(21.) By this rule we find how much any quantity in several denominations would amount to added to itself any number of times, and the easiest method is, manifestly, considering how

## COMPOUND DIVISION.

22. Here the divisor is a mere number, the dividend a quantity in several denominations.

Rule 1. Place the divisor in the same line with the dividend, with a line between them; divide the term of the highest denomination by the divisor; place the quotient in a line below; reduce the remainder to the next denomination, and add the quantity in that denomination; divide again, and continue till all are divided.

Ex. 1. Divide £20 6*s.* 7*d.* by 6.

| | £ | s. | d. |
|---|---|---|---|
| 6) | 20 | 6 | 7 |
| | £3 | 7 | $9\frac{1}{6}$ |

Rule 2. When the divisor is more than 12, we must find numbers not greater than 12 whose product will equal it, and divide by each successively.

Rule 3. Or divide as in Long Division: put the quo-

much each denomination would amount to repeated the required number of times, and writing all down, carrying the excess of each sum over that which can be written in that denomination to the next higher: but this amounts to the same thing as the rule.

(22.) Division is, finding a less quantity which is contained a required number of times in a given quantity. The rule separates the denominations into such parts as to admit in each denomination of finding numbers in that denomination contained in these parts the required number of times: thus, £20 6*s.* 7*d.* is separated into £18, £2 2*s.*, or 42 shillings, and 4*s.* 6*d.*, or 54 pence and one penny, because 3 pounds are contained exactly in 18 pounds, and no higher number of exact pounds can be found contained in 20 six times; 7 shillings are contained 6 times in 42; 9 pence 6 times in 54 pence; and there is a remainder of one. Hence we see, £3 7*s.* 9*d.*, with a remainder of one, is contained 6 times in £20 6*s.* 7*d.*, and we have worked exactly as the rule directs.

tient of the figures in the highest denomination, as the first term of the answer; bring the remainder to the next lower denomination, and add in the part in that denomination; divide again, placing this quotient as second term; reduce, and so continue till all are divided, reducing the last remainder lower if required. This Rule is universal.

Ex. 2. Divide £579 6*s.* 9*d.* by 18. 3. Also by 29.

```
Ex. 2.     £    s.  d.
       6) 579   6   9
          ----------
       3)  96  11  1½
          ----------
          £32   3  8½
          ----------
```

```
         £    s.  d.     £   s.   d.
    29) 579   6   9    (19  19   6½
         29
        ---
        289
        261
        ---
         28
         20
        ---
    29) 566 (19s.
         29
        ---
        276
        261
        ---
         15
         12
        ---
    29) 189 (6d.
        174
        ---
         15
          4
        ---
    29)  60 (2f. or ½d.
         58
        ---
          2
```

PROOF FOR DIVISION. Multiply the quotient by the divisor; add in the remainder,—the product must be the dividend.

PROOF FOR MULTIPLICATION. We may now prove multiplication by dividing the product by the multiplier,—this must give the multiplicand.

23. It must be particularly noticed that like quantities only can be added or subtracted from like quantities; we cannot add feet to shillings, or subtract one from the other.

Also, in Multiplication and Division, the multiplier and

divisor must be abstract numbers; we cannot multiply shillings by shillings,—such a thing as 8 shillings taken 3 shillings times can have no meaning.

---

## QUESTIONS ON THE COMPOUND RULES.

What is Reduction? What is meant by denomination? How can I reduce a quantity in one denomination to the next lower? How can I reduce a quantity of several denominations to one denomination? How can I bring a quantity in one denomination to others of a higher denomination? When must Rule 4 in division be used? How must perches be brought to yards? How yards to perches? What is Compound Addition? Are the quantities added together of the same kind? What is the rule? What is the proof? When the quantities are all equal, what other rule may be applied? What is Compound Subtraction? Are the quantities of the same kind? Give the rule. What is the proof? What is Compound Multiplication? What kind of number is the multiplier? Give the rule when the multiplier is less than 13. When the number is greater than 12, how must we proceed? What is the proof? What is Compound Division? What kind of number is the divisor? Give the rule when the divisor is less than 13. How must we proceed when the divisor is greater than 13? What is the proof in compound division? What proof may now be used in compound multiplication?

# CHAPTER III.

## ELEMENTARY PRACTICE.

24. Practice teaches how much any number of things will amount to at a given price each article.

Or if the price of any number of things be given, to find the price of each.

RULE 1. If the price be in one denomination, multiply it by the number of things, as in Simple Multiplication; bring the product to higher denominations, if necessary; the result will be the price of the whole.

RULE 2. If the price be in several denominations, multiply as in Compound Multiplication.

Ex. 1. Find the price of 60 yds., @ 3*d.* per yd.

```
    60 yards.
     3
   ----
12) 180
   ----
    15 shillings.
```

Ex. 2. 5 yds., @ at £3. 6*s.* 8*d.* per yd.

```
   £  s.  d.
   3  6   8
          5
  ----------
 £16 13   4
```

RULE 3. If the entire cost be known, and the price of each required, divide by the number, bringing the remainder into lower denominations, and dividing again.

Ex. 3. If 8 yds. cost £17, how much will one cost?

```
    £   s.  d.
8) 17   0   0
   ----------
   £2   2   6
```

Ex. 4. If 6 yds. cost £9 7*s.* 8*d.*, what is that per yd.?

```
   £   s.  d.
6) 9   7   8
  -----------
  £1  11   3⅓
```

---

(24) The reason of these rules is self-evident: if each one cost a certain price, any number of things must cost that number of times as much, and the reverse.

## SIMPLE PROPORTION.

(This is generally called Rule of Three.)

25. By this rule we are taught how to find a fourth quantity when three quantities are given (having a certain relation to each other), of which two only are of the same kind, and the third of the same kind as the required answer.

RULE. Place the quantity of the same kind as the required answer in the 3rd place.

If the answer will be greater than this, the greater of the other two terms must be placed 2nd.

If the answer will be less than this, the less must be placed 2nd. The other term must be placed 1st.

Reduce the 1st and 2nd terms to the lowest denomination mentioned in either, taking care to reduce them to the same denomination, and the 3rd term to the lowest denomination mentioned in it. Multiply the 2nd and 3rd terms together, and divide by the 1st, the quotient will be the answer, expressed in the denomination to which we have reduced the 3rd term, and must be raised to the highest denomination possible.

If the 1st term be greater than the 2nd, the 3rd will sometimes have to be reduced lower before dividing.

---

(25.) The reason of the rule may be given briefly as follows: when we have reduced to the same denomination, if we divide the price by the number of things bought, we get the price of each one thing, and multiplying by the number we wish to buy, gives us the price we want; now, if we multiply first, as in the rule, and divide last, it can make no difference in the answer. We reduce to the same denomination, since if, for instance, 1 inch cost 1 shilling, 3 yards would not cost 3, but 108 shillings; for each

The remainder must be reduced, if possible, and divided again.

Ex. 1. If 3 feet cost £1 7*s*. 9*d*., what will 5 feet 7 inches cost?

```
ft.   ft. in.    £  s. d.
 3  :  5   7 ::  1  7  9
12    12        20
--    --        --
36    67        27
                12
               ---
               333
                67
              ----
              2331
              1998
             -----   12
       36)   22311  (619
               216   ----
             ----- 20) 5,1  7
                71   ---------
                36   £2 11 7¾
               ---   ---------
               351
               324
               ---
                27
                 4
               ---
          36)  108  (3
               108
               ---
```

Here the answer will be in money, therefore £1 7*s*. 9*d*. is the last term; the number to be bought is greater, therefore the price or answer will be greater. Hence, 5 feet 7 inches must be in the 2nd term, and 3 feet the 1st; inches is the lowest name mentioned in either the 1st or 2nd, and therefore they must be reduced to inches; pence is mentioned in the middle term, and it must be reduced to pence.

Ex. 2. If 8 men do a piece of work in 6 days, how long will it take 4 men?

```
Men.  Men.    Days.
 4  :  8  ::   6
               8
              --
           4) 48
              --
              12 days.
              --
```

Here the answer will be in time, therefore, 6 days will be placed last; now since there are fewer men, and they do the same piece of work, they will be longer over it; hence, the answer will be greater than 6, therefore 8, the greater of the other two terms, must be placed second, and 4 will be first.

---

yard, being 36 inches, costs 36 shillings. In Ex. 1, 3 feet (or 36 inches) cost £1 7*s*. 9*d*. (or 333 pence); therefore one inch cost the 36th part of 333 pence, or 9¼*d*., and 5 feet 7 inches (or 67 inches) will cost 67 times as much, or £2 11*s*. 7¾*d*.

2. When there are any numbers which will divide both the 1st and 2nd, or the 1st and 3rd, they may be struck out.

We must take care, however, not to apply this rule to numbers which divide the 2nd and 3rd.

| | | |
|---|---|---|
| Thus— | 4 : 8 :: 6 | Now 4 divides both the 1st and 2nd. |
| Hence this becomes | 1 : 2 :: 6 | |
| | 2 | |
| | 12 days. | |

---

The reasoning in cases like the 2nd example will be as follows: As the work is the same, the labour employed must be the same; but if we reduce to the same denomination, the labour employed in the first case will be correctly estimated by the number of men multiplied by the number of days they work (thus, in the example we have 48 days' work of one man). Hence, I must find such a number, that multiplied into the number of men or days, it will give the same days' work (thus, 4 must be multiplied so as to give 48 days). But the product divided by the 3rd number will evidently give this (for the quotient multiplied by the divisor reproduces the dividend), and will be in the same denomination as the corresponding term of the same kind.

Reducing to the same denomination is absolutely essential; thus, if I am asked if 8 men do a piece of work in 2 months, how many men can do it in 15 days? $8 \times 2$ would represent months' work, but $15 \times$ a number, would only represent days' work, and, therefore, could not be equal to the same number of months' work.

2. The reason we can divide as stated, is, that in working we divide the product of the 2nd and 3rd terms by the 1st. But we may divide any divisor and dividend by the same number, and the quotient will still be the same. But since dividing the dividend alone would make the quotient less, it is evident we cannot strike out in the 2nd and 3rd alone and obtain the same result.

## QUESTIONS.

A 1. What does Practice teach? 2. How must the price of any number of things be found from that of one? 3. How of one from that of several?

B 1. What does Simple Proportion teach? 2. How many quantities are of the same kind? 3. Of what kind is the other? 4. Which quantity must be placed as the 3rd term? 5. Which quantity 2nd? 6. Will the 1st and 2nd terms be of the same kind? 7. To what denomination must they be reduced? 8. To what must the 3rd term be reduced? 9. Must it ever be brought lower? Which terms must be multiplied together? 10. By which term must we divide? 11. In what denomination will the quotient be? 12. Must the remainder be reduced lower?

C 1. When the price of a greater number of yards is required, will the price be greater? 2. Which will be placed as 3rd term? 3. Which as 2nd term?

D 1. When a greater number of men do the same piece of work as a smaller number, will the time be shorter? 2. What quantity will be placed 3rd? 3. What 2nd?

E 1. When the 1st and 2nd terms can be divided by the same number, how is it better to work? 3. Does the rule apply to the 1st and 3rd? 4. Does it apply to the 2nd and 3rd?

---

## BARTER.

26. Barter is a rule by which goods are exchanged for others of the same value.

---

(26.) The value of the goods to be received must be the same as those given; consequently we must find the number which

RULE. Find the value of the goods to be bartered: reduce this amount, and the price to be given, to the same denomination: divide by the latter, the quotient will be the answer.

When the price is required, divide by the number to be bought, reduced, if necessary, to one denomination.

Reference must be made to Art. 81, for an example, when none is given with the Rule.

## SIMPLE INTEREST.

27. This is the price paid for the use of money, and is generally reckoned at so much for a hundred pounds for one year, or so much per cent. per annum.

RULE 1. Multiply the principal (or sum lent) by the rate per cent. per annum, and divide by 100. This will give one year's interest.

RULE 2. Multiply this by the number of years. This will give the entire interest: adding the principal, we have the amount. When there is, besides, a part of

---

multiplied by the price of each article will give the same amount. But dividing the amount by the rate evidently gives the number, or dividing the amount by the number received gives the rate.

(27.) 1. The rate per cent. per annum is the interest of £100; therefore the rate per cent. divided by 100 is the interest for £1. 2. This multiplied by any number gives one year's interest for that number of pounds: multiplying by the number of years we get the entire interest. 3. Also the interest for one year divided by the number of days in the year gives the interest for one day. Multiplying by any number we get the interest for that number of days. 5. The interest for one year divided by the number of pounds is the interest for one pound. This multiplied by 100 will give the interest for £100, or the rate per cent.

a year, add the sum found by the following rule, for the time less than a year.

Rule 3. Reduce one year and the time to the same denomination: multiply the interest for one year by the time, and divide the product by the time in one year.

Rule 4. When the number of years is required. Divide the entire interest by that for one year.

Rule 5. When the rate per cent. is required. Divide the entire interest by the number of years. This will give one year's interest: multiply by 100, and divide by the principal: this will give the rate per cent.

Ex. Find the interest and amount of £330 for 3 years at 5 per cent. per annum.

Here £330 multiplied by 5 and divided by 100 gives £16 10*s.*, one year's interest; multiplying this by 3, we get £49 10*s.* as the entire interest. Adding the principal, £300, we find the amount £349 10*s.*

| | |
|---|---|
| | £ |
| | 330 |
| | 5 |
| 1,00 ) | 16,50 |
| | 16 10 |
| | 3 |
| | 49 10 |
| | 300 0 |
| | £349 10*s.* |

## COMMISSION, &c.

28. Commission or Brokerage is the rate per cent. given to an agent for the buying or selling of any article.

Insurance is a per centage paid for a sum to be given in case of loss by fire.

Rule. The rule is the same as that for Simple Interest for one year, namely, multiply the amount by the rate per cent. and divide by 100.

## STOCKS.

29. Stocks are government securities paying a certain rate per cent. From various causes they are often to be bought at prices different from the nominal value.

RULE 1. To find how much must be given for a certain amount of Stock, at a stated price per hundred. Multiply the amount to be purchased by the rate per hundred, and divide by 100.

RULE 2. To find how much Stock a given amount will purchase. Multiply the given sum by 100, and divide by the rate per cent.

RULE 3. To find what will be the rate per cent. per annum obtained by purchasing Stock at a given price. Multiply the rate per cent. per annum of the Stock by 100, and divide by the price given.

To find the income derived, we must multiply by the amount invested, instead of 100.

---

(29.) Rule 1. The price per hundred divided by one hundred gives the price for one pound's worth of Stock, and this multiplied by the number to be purchased gives the price for that amount of Stock. 2. One hundred pounds of Stock divided by the price per hundred is the amount one pound will purchase (for this multiplied by the price of one hundred will give £100 of Stock); therefore, multiplying by the sum to be invested, we have the amount of Stock for that sum. 3. The rate per cent. per annum of the Stock is the interest for the price given for 100; therefore, dividing by the price given we find the yearly interest of one pound, this multiplied by 100 gives the interest for one hundred invested, or by any other number of pounds, gives the yearly interest for that sum invested.

## PROFIT AND LOSS.

30. The Profit or Loss on selling goods, bought at a given price and sold at another, is calculated by the following rules.

RULE 1. To find the Profit or Loss per cent. Find the Profit or Loss on the amount given. Multiply it by 100, and divide by the amount.

RULE 2. To find how goods must be sold to gain so much per cent. Multiply the price at which they are bought by 100, together with the rate per cent., and divide by 100.

---

## PARTNERSHIP.

31. When two or more persons are in partnership for the same length of time, their respective gain is computed by the following rule.

RULE. Multiply the entire gain by each one's share,

---

(30.) The Profit and Loss on any amount, divided by that amount, gives the Profit or Loss on one pound (or if the amount be expressed in a lower denomination, of one of that denomination), and multiplied by 100 (or 100 expressed in that denomination) gives the Profit or Loss on £100. Rule 2. £100, and the profit on it, is the price at which one hundred pounds' worth must be sold; this divided by 100 gives the price of which one pound must be sold (or divided by 100 expressed in any denomination, what the worth of one in that denomination must be sold at), therefore, multiplying by the price at which they were bought, we shall get how much that amount should be sold at.

(31.) The entire gain on any amount, divided by that amount, gives the gain on £1, and this multiplied by any number of pounds gives the gain on that number.

Compound Partnership is explained in Art. 76.

and divide by the sum of all the shares: this will give each partner's gain.

*This is called Simple Partnership.*

When the time is different.

Rule. Reduce the time to the same denomination; multiply the sum each one has in the firm by the time he had it employed; add all these products together; multiply the entire gain by each of the products, and divide by the sum of them to find the respective shares.

*This is called Compound Partnership.*

*In cases of Bankruptcy*

Add all the debts together; divide the assets by them; this will give the rate per £, and, multiplied by each man's debt, will give what must be paid to him.

---

## DISCOUNT.

32. Discount is the abatement made for money paid before it is really due, and is such that the sum paid would amount at interest, in that time, to the sum due.

Rule. Multiply the principal by the interest for £100 for the time given, and divide by 100 with that interest; this will be the Discount;—subtract it from the sum; this will give the present worth.

Discount is generally calculated as interest, and not by the above rule.

(32.) The interest of £100 for the time given is the Discount on £100 with that interest; therefore, dividing by 100 with its interest, we get the Discount of £1 for the same time, and multiplying this by any number, we get the Discount on that number of pounds.

## COMPOUND INTEREST.

33. When the interest at the end of each year is added to the amount, and the whole bears interest, it is called Compound Interest.

**Rule.** 1. Find the interest for the first year, and add it to the principal. Find the interest of this sum for the second year, add it to the sum, and so continue for the whole number of years; this will give the entire amount. Subtract the principal; this will give the Compound Interest.

**Rule** 2. When the time is not an exact number of years, find what the principal and accumulated interest will amount to for the number of complete years, then find the amount of this sum, at Simple Interest, for the part remaining.

Ex. What are the Compound Interest and amount of £300 in 3 years, at 5 per cent.?

```
                £
               300
                 5
      1,00)  15,00
            £15       Interest for the 1st year.
            300
           £315 ...  Amount at the end of the 1st year.
              5
    1,00)  15,75
          £15 15s. Interest for the 2nd year.
          315
        £330 15s. Amount at the end of the 2nd year.
               5
 1,00) 16,53 15
      16  10  9 Interest for the 3rd year.
     330  15  0
     347   5  9 Amount at the end of the 3rd year.
     300
    £47    5  9 Entire Interest.
```

When the interest is payable at shorter intervals we must calculate the interest for the first interval, add it to the principal, calculate for the second interval, add again, and so on for the entire number of intervals.

---

## EQUATION OF PAYMENTS.

34. This is when a sum has to be paid at different times, and it is required to find a time when the whole may be paid.

RULE 1. Multiply each sum by the time between its payment and that of the last payment (reducing the time to the same denomination). Add all these products together, divide by the sum to be paid: this will give the interval between the equated time and the last payment.

Equation of Payments for Compound Interest is difficult to calculate, but the result would be given correctly by finding the present value.

RULE. Find the present value of each of the sums, and calculate the time when the sum of the present values, at interest, would amount to the entire sum to be paid.

This rule applies equally to Simple Interest.

---

35. It will be well to observe that almost all the rules which have preceded are mere modifications of the rule for Simple Proportion, and may be worked in the same manner.

In Barter, the number of articles, though of a completely different nature, must be stated as things of the same kind, each being reduced to one name.

In Simple Interest, Commission, and Discount, the interest or rate given, and that required, must be considered as things of the same kind. The interest, or the amount of one year, must first be found, and next that for the required period.

In Stocks, the two amounts of stock will be things of the same kind.

In Profit and Loss, the profit on the price, and that on £100.

In Partnership, the gain of each partner, and the entire gain.

When the three given quantities are different sums of money, it is generally better to place the second term third, if it be of higher denomination than the first and third, or to reduce the first and third terms only to the lowest denomination, the quotient will be in the higher denomination.

Ex. 1. The interest for 1 year being £15 12*s.*, find that for 9 weeks.

```
wks.   wks.      £  s.
 52  :  9  ::   15 12
                20
               ----
               312
 13  :  9  ::   78 Dividing 1st and 3rd by 4.
                 9
               ---
          13 ) 702 ( 54s.
               65   ----
               --- £2 14s.
                52
                52
               ---
```

## QUESTIONS.

A 1. What is Simple Interest? 2. What is the principal? 3. What the amount. 4. What is the rule for finding the entire interest and amount. 5. When there is a part of a year over, what is the rule? 6. How must the rate per cent. be found? 7. How must the number of years be found?

B 1. What is Discount? 2. How is it generally found?

C 1. What is Compound Interest? 2. How must the amount and interest be found? 3. When the time is not an exact number of years, how must the interest and amount be found? 4. How may the rule of Simple Proportion be applied to Simple Interest and Discount? 5. When may the third term be placed second?

A 1. What is Barter? 2. Give the Rule. B 1. What is Commission? 2. What Insurance? 3. How must we calculate either? C. 1. What are Stocks? 2. How must the price of a given amount of Stock be calculated? 3. How must we find how much we can buy for a given sum? 4. How must the rate per cent. to be obtained by buying stocks, at a given price, be found? D 1. What is meant by Profit and Loss? 2. How must the Profit or Loss per cent. be calculated? 3. How must the price at which goods must be sold to gain so much per cent. be found? E 1. What is Simple Partnership? 2. How must the gain of each partner be found? 3. What is Compound Partnership? 4. How must the time be expressed? 5. How must the gain of each partner be found? 6. How must the rate per £ in bankruptcy be calculated? 7. How what each creditor must receive? F. 1. What is Equation of Payments? 2. Give the rule for finding the Equated time.

# CHAPTER IV.

## GREATEST COMMON MEASURE.

**36.** A factor, or a measure of a number, is any number which will exactly divide it, or is contained in it a certain number of times.

A common measure of two or more numbers, is a number which will exactly divide each of them.

The greatest common measure of two or more numbers, is the greatest number which will divide both or all of them without remainder.

---

(36.) The *reason of this operation is as follows:—The greatest common measure cannot be greater than the smaller of the two numbers, as it must divide both. We divide the greater by the less to see if it will exactly divide it, for then the less will manifestly be the greatest common measure.

---

* It may make the proof given here clearer if the following propositions be read:—

1. A number which divides another will divide it when it is multiplied by any other. For the increased number is only the original number added to itself a certain number of times (11), but the original number itself is the common measure added to itself (12); hence the increased number must be also the measure added to itself, or the measure must divide it.

2. A common measure of two numbers will measure their sum or difference; each number is the common measure repeated, therefore, adding the numbers is adding the common measure; hence the sum will be the common measure added to itself, and therefore be measured by it. Likewise subtracting one from the other is subtracting the common measure a certain number of times, but not as often as we can, since we could subtract it till there was no remainder: therefore, the remainder, that is their difference, must always be such that we can subtract the common measure till we destroy it, therefore it is divisible by it.

Hence we may multiply two numbers by any others, then add or take their difference: their measure will divide all the numbers thus formed.

RULE 1. Divide the greater of the two numbers by the less; bring down the less and place it in the same line with the remainder (if any), divide it by that remainder, if there be a second remainder, bring down the first for a dividend, and use the second as a divisor, and continue till there is no remainder, the last divisor will be the greatest common measure.

```
Ex. 388)948(2
        776                         388 ÷ by 4 is 97
        ---                         948 ÷ by 4 is 237
        172)388(2                     97 and 237 have no
            344                       common measure.
            ---
             44)172(3
                132
                ---
                 40)44(1
                    40
                    --
Therefore 4 is the greatest  4)40(10
  common measure.              40
                               --
```

RULE 2. When there are more than two numbers, find the greatest common measure of the first two, then the greatest common measure of this and the third, and

---

There being a remainder shews the less cannot be the greatest common measure, but the number must be smaller still. Now (I.) every common measure of the two first number, is a measure of this remainder. For, as division is merely subtraction repeated in any common measure of the divisor and dividend, the divisor (which is the common measure added sufficiently to itself) subtracted any number of times, is the same as this common measure subtracted a number of times greater in proportion; but since the common measure divides exactly the dividend, the remainder will be such that the common measure subtracted from it a proper additional number of times will destroy it—hence it must divide it; therefore, every common measure of the two orginal numbers will be a common measure of the first remainder.

proceed in like manner. The last greatest common measure will be that of all the numbers.

RULE 3. Sometimes a shorter method may be used. Divide the numbers by 2 as many times as possible (which will be till one ends in an odd number), next try if we can divide by 3, next by 5, and by 7, &c.; multiply all the divisors together, they will give the greatest common measure.

We must be careful to observe that the last quotients have no common factors. Rule 1. must be applied to them when there is any doubt.

---

II. Also every common measure of this remainder and the less number, will be a common measure of the greater: for the remainder and the less number added to itself a certain number of times, equal the greater (as multiplication is merely addition, see 11), and both these being only any one of their common measures added together, it follows that any of their common measures added to itself often enough will give the greater, or be a measure of the greater. Therefore from (I. and II.) their common measures must be identical, and the greatest of any two of them will evidently be the greatest common measure of the three.

So we have now to find merely the greatest common measure of two numbers, the greater of which is the less in the first case, or the process is equivalent to reducing the numbers to the lowest form possible; so dividing the smaller number by the first remainder, we can shew, in the same manner, that the greatest common measure of the next remainder and the first remainder, is the G. C. M. of the first divisor and the first remainder, and, therefore, by (I. and II.) of the two original numbers; proceeding in the same way, we arrive at the same conclusions, that the greatest common measure of two consecutive remainders is the greatest common measure sought, and of course can never exceed the latter remainder; but at last we arrive at a remainder which exactly divides the former, and it must therefore be the greatest common measure.

## LEAST COMMON MULTIPLE.

**37. A multiple of a number is one which contains it any number of times.**

**The least common multiple of two or more number is the least number which is a multiple of them**

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To illustrate this from the example, any common measure of 388 and 948 will be a common measure of 948, and 2 × 388, or 776, and therefore of their difference, 172 (since we can subtract the common measure from 984 till there is no remainder; but 776 being the common measure added to itself (12), subtracting it from 948 is the same as subtracting a greater number of times the common measure)—and, therefore, of 388, and 2 × 172, or 344, and therefore of their difference, 44; and similarly of the difference of 172 and 3 × 44, which is 4. That is, any common measure of 388 and 948 is a common measure of any of the remainders in the operation, and cannot, therefore, be greater than the last 4. But 4 is a common measure, for it divides 40 and itself, or 44, and therefore 3 × 44 and 40, or 172, and therefore 2 × 172 and 44, or 388, and therefore 2 × 388 and 172, or 948. For all these numbers are made up by adding four.

Hence it is a measure of both 948 and 388; and as no measure is greater than it, it must be the greatest.

3. This will evidently give the G. C. M., for we divide the numbers by all of the divisors, that is, by a number equal to their product; and, as we can divide by no other, the quotients have no common factor, or we have divided by the greatest common measure.

(37.) The reason of the rules is the following:—The two numbers multiplied together will give a common multiple, but not generally the least. The greatest common measure is all that is common to both, and this product is the factors, which have no common measure, multiplied together × greatest common × greatest common measure, since the greatest common measure is contained in both numbers. Dividing, as in the rule, we shall have factors having no common measure multiplied together × the greatest common

respectively; or the least number which can be divided exactly by all of them separately.

RULE 1. When there are two numbers, find their greatest common measure; divide one of the numbers by it, and multiply the quotient by the other; this will give the least common multiple.

RULE 2. When there are more than two numbers, find the least common multiple of the two first; then of this multiple and the third, and so on till we have found it for the last. This will be the least common multiple of all.

RULE 3. There is another method sometimes shorter than this when there are many numbers. Place all the numbers in a line, strike out any which are exactly contained in another not struck out; divide those not struck out by any number common to two or more; place the quotient and the numbers remaining in a line below; strike out and proceed as before; multiply the numbers

---

measure of the two original numbers. This product will produce each number multiplied by the part in the other, having no common factor, and be a common multiple, and also the least, as we can exclude nothing more.

3. This product will be the least common multiple, because in striking out we always keep in a number which contains or is a multiple of that struck out, ∴ the numbers multiplied together in any line, contains also the numbers struck out in that line. So the striking out the numbers leaves the L. C. M. the same: but all that it is common besides are the divisors, and as the L. C. M. need only contain once the number which divides several, we have merely to multiply the product of the quotients by the common divisors to find a number which is a multiple, and is the least, because there is nothing more which can be excluded.

in the final line and all the divisors together. This will give the least common multiple.

Ex. Find the L. C. M. of 156 and 208

```
156) 208 (1
     156
     ---
      52) 156 (3
          156
          ---
```

156 ÷ by 52 is 3, and 3 × 208 gives the L. C. Multiple.

Find the L. C. M. of 4, 20, 16, 5, 9, 3, 8, 12

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 4) | (4) | 20 | 16 | (5) | 9 | (3) | (8) | 12 | 15 |
| 3) | | (5) | | 4 | | 9 | | (3) | 15 |
| | | | | 4 | | 3 | | 5 | |

Least common multiple is 5 × 3 × 4 × 3 × 4, or 720.

## QUESTIONS.

A 1. What is a Factor or measure of a number? 2. What is a Common Measure of two or more numbers? 3. What is the Greatest Common Measure? 4. How must it be found for two numbers? 5. How must it be found for more than two numbers? 6. What other rule is sometimes best to be applied?

B 1. What is a Multiple of a number? 2. What is the Least Common Multiple of two or more numbers? 3. How must it be found from the G. C. M. of two numbers? 4. How for more than two numbers? 6. What other rule may be used when there are many numbers?

# CHAPTER V.

## FRACTIONS.

38. A *fraction* is a quantity representing a part or parts of a unit or whole.

The word Fraction itself denotes something broken off, not a whole; so that in numbers it means a part or parts of one or a whole.

It consists of two numbers, with a line (representing division) between them. The upper one is called the numerator, and the lower the denominator, as $\frac{2}{3}$, where the 2 is the numerator, 3 the denominator.

A fraction may also be thus defined. A fraction is a quantity whose denominator represents into how many parts the unit or whole is divided, and the numerator how many of those parts are taken or added together.

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(38.) It is evident, from this second definition, that when the denominators are the same, the value of the fraction is shewn by the numerator, and their comparative value depends entirely upon it.

Thus, in $\frac{1}{7}$, $\frac{3}{7}$, $\frac{6}{7}$. Here the whole is divided into 7 parts, $\frac{1}{7}$ denotes one seventh part, $\frac{3}{7}$ denotes 3 of them, and is as 3 times as great as $\frac{1}{7}$, or is $\frac{1}{7}+\frac{1}{7}+\frac{1}{7}$; $\frac{6}{7}$ denotes 6 of them, and is therefore 6 times as great as $\frac{1}{7}$, twice at great as $\frac{3}{7}$, or is $\frac{3}{7}+\frac{3}{7}$.

Observe, $\frac{3}{7}$ is also the same as $\frac{1}{7}$ of 3; $\frac{6}{7}$ as $\frac{3}{7}$ of 3, or $\frac{1}{7}$ of 6.

And, as the numerators represent the number of parts added together, it follows that to add fractions of the same unit, having a common denominator, we have merely to add the numerators, or subtract the numerators in subtracting the fractions, placing, in each case, the common denominator below the result. And, therefore, when the denominators are different, if we reduce the fractions to equivalent fractions, with the same denominator, we may then apply to the numerators the same rules as in addition or subtraction of whole numbers.

Thus, in $\frac{2}{3}$ the unit is divided into three parts, and two of them are taken; taking three parts, the fraction would become $\frac{3}{3}$, and, as we see, we have taken as many parts as make the whole.

It follows that the unit and a fraction with a numerator and denominator the same, are identical.

Similarly $\frac{2}{3}$ of 4 denotes that 4 is to be divided into three parts, and two of them taken; $\frac{3}{3}$ of 4 would mean that 3 are to be taken, or a whole, that is the entire number—4.

A *proper fraction* is when the numerator is less than the denominator.

An *improper fraction* is when the numerator is greater than the denominator, or when we take more parts than make up a whole.

*Mixed numbers* consist of a whole number and a fraction, and can always be represented by improper fractions.

Thus $\frac{2}{3}$ is a proper fraction; $\frac{4}{3}$ an improper fraction; $1\frac{1}{3}$ a mixed number.

A *simple fraction* is a fraction standing alone and representing a part or parts of a whole, as $\frac{2}{3}$.

A *compound fraction* is a fraction of a fraction, or parts, not of a whole but of a fraction, as $\frac{1}{2}$ of $\frac{3}{5}$; $\frac{1}{2}$ of $\frac{3}{5}$ of $\frac{7}{8}$.

A *complex fraction* is one whose numerator and denominator are not both whole numbers, as $\dfrac{3\frac{1}{3}}{4}$ $\dfrac{3}{4\frac{1}{3}}$ $\dfrac{3\frac{1}{4}}{4\frac{1}{3}}$.

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## REDUCTION OF FRACTIONS.

39. RULE 1. To reduce fractions to their lowest terms. Find the greatest common measure of the numerator and denominator, and divide them by it.

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(39.) In Ex. 1 the whole number was divided into 27 parts; our new denominator is 9, that is, the whole number is now divided into 9 parts only, or the parts are three times as large. We

Ex. Reduce $\frac{39}{27}$ to its lowest terms.

```
27) 39 (1        39 ÷ by 3 is 13
    27           27 ÷ by 3 is  9     13/9 is the fraction.
    ——
    12) 27 (2
        24
        ——
         3) 12 (4
            12
            ——
```

Rule 2. To express whole numbers by fractions with any required denominator. Multiply the whole number by the required denominator, the product will be the numerator.

Ex. Express 7 as a fraction with denominator 6.

$7 \times 6$ is 42, therefore the fraction is $\frac{42}{6}$.

Hence, if the numerator of a fraction be divisible by the denominator, the quotient will be a whole number equal to the fraction, and a whole number may be written as a fraction, with 1 for its denominator.

---

must therefore take a third of the number we took in the former case, to get the same value; or, we must divide the old numerator by 3, the number by which we divided the denominator.

In Ex. 2 the unit is to be divided into 6 parts; 6 of these will make one unit; 42 will manifestly make 7 units.

In Ex. 3, since a fourth part is three times as large as a twelfth part, we must take three times as many of the latter, as of the former, to get the same value, or multiply the numerator by the same number as we have multiplied the denominator.

To see the correctness of the rules, we must remember that the denominator represents into how many parts the whole number is divided; therefore, if we divide or multiply the denominator by any number, the reduced or increased denominator shews that now the whole number is divided into fewer or more parts, and as, therefore, the parts are larger or smaller, we must take fewer or more of them, that is, diminish or increase the numerator in proportion.

**Rule 3.** To express a fraction by another fraction with a required denominator. Divide the required denominator by the first, and multiply the numerator by the quotient.

Ex. Express $\frac{3}{4}$ as a fraction whose denominator is 12.

Here 12 divided by 4 gives 3; and $3 \times 3$ is 9, therefore the fraction is $\frac{9}{12}$. This is the same as multiplying the numerator and denominator by the same number.

These three rules shew that the numerator and denominator may be multiplied or divided by the same number without altering the value of the fraction.

40. To reduce fractions to others having a common denominator.

**Rule.** Find the L. C. M. of all the denominators, this will be the common denominator. Divide this by the first denominator, multiply the first numerator by the quotient: the product will be the new numerator; proceed in the same way with all the fractions, and write the common denominator under the new numerators.

Ex. Reduce $\frac{5}{7} + \frac{1}{2} + \frac{3}{4} + \frac{2}{3} + \frac{3}{14} + \frac{1}{6}$.

Here the L. C. M. (found by 37, Rule 3) is 84, and 84 $\div$ by 7 is 12, and $5 \times 12$ is 60, $\because$ $\frac{5}{7} = \frac{60}{84}$; 84 $\div$ by 2 is 42, and $1 \times$ by 42 is 42, $\because$ $\frac{1}{2}$ is $\frac{42}{84}$. Similarly $\frac{3}{4} = \frac{63}{84}$, $\frac{2}{3} = \frac{56}{84}$, $\frac{3}{14} = \frac{18}{84}$, $\frac{1}{6} = \frac{14}{84}$, therefore $\frac{60}{84}$, $\frac{42}{84}$, $\frac{63}{84}$, $\frac{56}{84}$, $\frac{18}{84}$, $\frac{14}{84}$, will be the fractions reduced to a common denominator.

---

(40.) To reduce fractions to the same denominator, the least possible number should be found, but it must include all the denominators, and therefore cannot be less than the L. C. M. of them. Now the first denominator multiplied by the number we have multiplied the numerator (that is the L. C. M. divided by the first denominator), will manifestly give the L. C. M.

Similarly all the other fractions will have this number for the denominator, and (by 39) their value will not be altered. Hence the correctness of the rule.

41. **Rule 1.** To reduce improper fractions to whole or mixed numbers. Divide the numerator by the denominator, the quotient will be the whole number, the remainder the numerator of the fractional part.

Ex. Reduce $\frac{23}{4}$ to a mixed number.

$$\begin{array}{r} 4)\ 23\ (5, \text{ and } \because \frac{23}{4} = 5\frac{3}{4} \\ 20 \\ \hline 3 \end{array}$$

**Rule 2.** To reduce mixed numbers to improper fractions. Multiply the whole number by the denominator, add the product to the numerator, the result will be the new numerator.

Ex. Reduce $5\frac{3}{4}$ to an improper fraction.

$5 \times 4$ is 20, and $20 + 3$ is 23

therefore $5\frac{3}{4} = \frac{23}{4}$.

**Rule 3.** To reduce complex fractions to simple ones. Reduce the numerator and denominator to simple fractions if necessary. Multiply each fraction by the product of the two denominators, and reduce to lowest terms.

Ex. Reduce $\dfrac{2\frac{1}{3}}{3}$ $\dfrac{2}{3\frac{1}{3}}$ $\dfrac{2\frac{1}{3}}{3\frac{1}{4}}$ to simple fractions.

$$\frac{2\frac{1}{3}}{3} = \frac{\frac{7}{3}}{3} = \frac{\frac{7}{3} \times 3}{3 \times 3} = \frac{7}{9} \qquad \frac{2}{3\frac{1}{3}} = \frac{2}{\frac{10}{3}} = \frac{2 \times 3}{\frac{10}{3} \times 3} = \frac{6}{10} = \frac{3}{5}$$

$$\frac{2\frac{1}{3}}{3\frac{1}{4}} = \frac{\frac{7}{3}}{\frac{13}{4}} = \frac{\frac{7 \times 4 \times 3}{3}}{\frac{13 \times 4 \times 3}{4}} = \frac{7 \times 4}{13 \times 3} = \frac{28}{39}$$

In practice, after reducing to simple fractions, multiply the extremes for numerator, and the means for the denominator.

---

(41.) $\frac{23}{4}$ represents 23 fourths—but 4 fourths make a whole; therefore 23 fourths equal 20 fourths and 3 fourths, or 5 and 3 fourths—$5\frac{3}{4}$. The converse exactly applies in the next case.

The general reasoning will be, an improper fraction results

## ADDITION OF FRACTIONS.

42. RULE 1. Reduce all the fractions to simple ones if necessary; multiply all the denominators for a new denominator, and each of the numerators by all the denominators except its own; add them together, place the common denominator under the sum, and reduce to the lowest terms.

Ex. Add $\frac{3}{4} + \frac{2}{3} + \frac{5}{6} + \frac{1}{2}$.

$$\begin{array}{lll}
4 \times 3 \times 6 \times 2 = 144 & \text{the new denominator.} & \\
3 \times 3 \times 6 \times 2 = 108 & \text{the first num.} & \text{Fraction is } \frac{396}{144} = \frac{11}{4} = 2\frac{3}{4} \\
2 \times 4 \times 6 \times 2 = \phantom{0}96 & \text{the second num.} & 144)\ 396\ (2 \\
5 \times 4 \times 3 \times 2 = 120 & \text{the third num.} & \phantom{144)}\ 288 \\
1 \times 4 \times 3 \times 6 = \phantom{0}72 & \text{the fourth num.} & \phantom{144)}\ \overline{108)\ 144\ (1} \\
\phantom{1 \times 4 \times 3 \times 6 =}\ \overline{396} & & \phantom{144)\ 108)}\ 108 \\
 & & \phantom{144)\ 108)}\ \overline{36)\ 108\ (3} \\
 & & \phantom{144)\ 108)\ 36)}\ 108
\end{array}$$

This rule is only of use when the denominator has no common factors.

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from taking more parts than make up a whole number; if the number of parts taken make several whole numbers from (39, 2) the numerator divided by the denominator will give the number, and the remainder, when this is not the case, shews there are besides some parts less than a whole number.

In Rule 3 the first step is taken by Rule 2; the second, because we can multiply the numerator and denominator of a fraction by the same number.

(42.) It has already been shewn (39) that we can multiply or divide the numerator and denominator by the same number, without altering the value of the fraction. And from what has been said before (38) it is evident, when fractions have the same denominator, their numerators may all be added together or subtracted, as they represent numbers of the same parts.

Thus, in Ex., Rule 2, we have 9 twelfths, 8 twelfths, 10 twelfths, and 6 twelfths; and adding together would be merely adding the number of twelfths, that is, the numerators.

**Rule 2.** Reduce to simple fractions if necessary. Find the least common multiple of the denominators, and write it as the new common denominator. Reduce all the fractions so as to have this denominator (40), add the numerators, and reduce the final fraction to its lowest terms.

Ex. Add $\frac{3}{4} \times \frac{2}{3} \times \frac{5}{6} \times \frac{1}{2}$.

Here 12 is the L. C. M. of the denominators,

And $\frac{3}{4} = \frac{9}{12}, \quad \frac{2}{3} = \frac{8}{12}, \quad \frac{5}{6} = \frac{10}{12}, \quad \frac{1}{2} = \frac{6}{12}$

adding, we obtain $\frac{9 + 8 + 10 + 6}{12} = \frac{33}{12} = \frac{11}{4} = 2\frac{3}{4}$.

## SUBTRACTION OF FRACTIONS.

43. **Rule.** Reduce the fractions to others having a common denominator, take the difference of the numerators as in simple subtraction, and reduce the resulting fraction to its lowest terms.

Ex. Find the value of $\frac{3}{4} - \frac{2}{3} + \frac{5}{6} - \frac{1}{2}$.

Reducing to a common denominator, we obtain .................. $\frac{9 - 8 + 10 - 6}{12} = \frac{5}{12}$

## MULTIPLICATION OF FRACTIONS.

44. **Rule 1.** First by a whole number. Multiply the numerator by the whole number.

---

(44.) Multiplying a fraction by a fraction must mean that we must not multiply by the numerator, but must multiply by the quantity which is the result of the division of the numerator by the denominator. Therefore, if we multiply by the numerator alone, we shall have multiplied by too much by that number of times represented by the denominator, that is, we ought now to divide by the denominator to get the proper value. But in

For "multiply" means "add to itself," and to add a fraction to itself (by 42), we must add the numerator to itself.

Ex. Find the value of $\frac{3}{5} \times 4$.

Here, $3 \times 4 = 12$ ∴ $\frac{3}{5} \times 4$ is $\frac{12}{5}$ or $2\frac{2}{5}$.

Rule 2. By a fraction or fractions. Multiply the numerators together for a new numerator, and the denominators for a new denominator.

Ex. $\frac{2}{3} \times \frac{3}{4}$ $\qquad \frac{2 \times 3 \text{ is } 6}{3 \times 4 \text{ is } 12}$ ∴ $\frac{6}{12}$ or $\frac{1}{2}$ is the answer.

When any of the numerators have common factors with the denominators, we may strike them out before we multiply.

Under this rule come all such expressions as $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{2}{5}$, or compound fractions.

Ex. Find the value of $\frac{3}{4}$ of $\frac{2}{3} + \frac{5}{12}$ of $\frac{3}{4} \times \frac{4}{5}$.

$$\frac{3}{4} \text{ of } \frac{2}{3} = \frac{6}{12} \text{ or } \frac{1}{2}$$

$$\frac{5}{12} \text{ of } \frac{3}{4} \times \frac{4}{5} \text{ is } \frac{5 \times 3 \times 4}{12 \times 4 \times 5} = \frac{60}{240} \text{ or } \frac{1}{4}$$

and $\frac{1}{2} + \frac{1}{4}$ is $\frac{2}{4} + \frac{1}{4}$ or $\frac{3}{4}$. It is better at once to strike out

$$\frac{(3)}{4} \times \frac{2}{(3)} = \frac{2}{4} = \frac{1}{2}, \ \frac{(5)}{12} \times \frac{3}{(4)} \times \frac{(4)}{(5)} = \frac{3}{12} = \frac{1}{4}$$

---

multiplying a fraction by a number, we multiply the numerator; in dividing a fraction by a number (by 45, Rule 1), we multiply the denominator. Hence the rule, and, if the multiplying fraction be a proper one, the multiplier will then be less than the divisor; therefore, we shall get a smaller fraction than the first.

A fraction of a whole number denotes that we must take that number of parts of it the fraction is of unity—this is shewn by multiplying the numerator.

A fraction of a fraction, as $\frac{2}{3}$ of $\frac{1}{4}$, expressed in words, will mean "divide $\frac{1}{4}$ into three parts, and take two of them," but a fourth, divided into three parts, will give twelfths, and two twelfths is

## DIVISION OF FRACTIONS.

45. **Rule** 1. By a whole number. Multiply the denominator by it.

Ex. Divide $\frac{2}{3}$ by 5. $3 \times 5 = 15$, and answer is $\frac{2}{15}$.

**Rule** 2. By a fraction. Multiply the numerator of the fraction to be divided by the denominator of the

---

written $\frac{2}{12}$. Again, $\frac{2}{3}$ of $\frac{3}{4}$ is, divide $\frac{3}{4}$ into 3 parts, and take two of them; as before, a fourth into three parts gives twelfths. But we were given $\frac{3}{4}$ to divide, or a part three times as great, so the subdivided parts will be three times as great, or will be each 3 twelfths, but we are required to take 2 of three twelfths, or $\frac{6}{12}$, that is $\frac{1}{2}$. So in every case the denominators multiplied together will denote the number of subdivisions; and the numerators, multiplied, will give the number of subdivisions to be taken.

(45.) Division by a whole number is equivalent to subdividing the parts; hence we multiply the denominator. Division by a fraction means, that if we divided by the numerator only, we shall have divided by too much, but must only divide by that part resulting from its division by its own denominator; therefore, if we divide by the whole numerator, we must multiply by the denominator, as we have divided that number of times too much. This gives the rule, and we shall always get a greater fraction when the dividing fraction is a proper one.

It thus appears that multiplying by a proper fraction diminishes, dividing by a proper fraction increases the number. This may be made clearer by considering that a proper fraction is less than one; and that, as the multiplier diminishes, so must the product. Now, multiplying by more than unity increases the number; by unity, leaves it just the same, and it seems natural to expect that multiplying by less than one should diminish, or should be really division. Similarly diminishing the divisor increases the quotient, and when the divisor is one, the quotient is the dividend: so it appears that dividing by less than one is really multiplying.

divisor, and its denominator by the numerator; that is, invert the divisor, and proceed as in multiplication.

Ex. Divide $\frac{3}{4}$ by $\frac{2}{3}$.

$$\tfrac{3}{4} \div \tfrac{2}{3} = \tfrac{3}{4} \times \tfrac{3}{2} = \tfrac{9}{8} = 1\tfrac{1}{8}.$$

When the divisor or dividend is composed of several fractions, each must be expressed as one fraction.

Ex. Divide $(3\frac{1}{2} + \frac{5}{3} \text{ of } \frac{3}{4})$ by $(\frac{2}{3} + \frac{1}{4} \times \frac{3}{2})$.

$$\text{Here} \quad 3\tfrac{1}{2} + \frac{5}{3} \text{ of } \frac{3}{4} = \frac{7}{2} + \frac{5}{4} = \frac{14 + 5}{4} = \frac{19}{4}$$

$$\text{and} \quad \frac{2}{3} + \frac{1}{4} \times \frac{3}{2} = \frac{2}{3} + \frac{3}{8} = \frac{16 + 9}{24} = \frac{25}{24}$$

$$\text{and} \quad \frac{19}{4} \div \frac{25}{24} = \frac{19}{4} \times \frac{24}{25} = \frac{19 \times 6}{25} = \frac{114}{25} = 4\tfrac{14}{25}$$

---

## REDUCTION OF FRACTIONS OF CONCRETE QUANTITIES.

46. **Rule** 1. To find the value of any fraction, a given quantity of one, or mixed denominations. Multiply the quantity by the numerator of the fraction, and divide the product by the denominator; reduce the remainder, if

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(46.) Fractions of concrete quantities will be in the same denomination as the unit (or of the whole expressed in any denomination when we have multiplied the numerator), and must be multiplied by the number of the next lower denomination in the higher, to be expressed in that denomination. But to multiply fractions we must multiply the numerators (44).

Now it is clear that a fraction of a higher denomination may contain several units of a lower, so dividing by the denominator, the result will be in the denomination to which we have reduced the numerator: thus £$\frac{5}{8}$ will be $\frac{100}{8}$ shillings, and $\frac{3}{4}$ £3 10*s.* must be $\frac{3}{4}$ of 70*s.*, or $\frac{210}{4}$ shillings.

2. To reduce one quantity to the fraction of another: it is evident that the fraction which the first quantity is of the second,

required, to lower denominations, and divide again by the denominator.

Ex. Find the value of $\frac{5}{6}$ of £1; also $\frac{3}{4}$ of £4 10*s*.

```
    20                                £  s.
     5                                4  10
   ----                                   3          £ s. d.
6) 100              s. d.          -------  Answer 3  7  6
   ----
    16 -  4   Answer 16  8.     4)  13  10
         12                        -------------
         --                        £3   7   6
     6)  48                        -------------
         --
          8
```

**Rule 2.** To express any quantity as the fraction of another. Reduce to a common denomination; write the quantity to be expressed as a fraction for numerator, and the other as denominator.

Ex. Reduce 1*s*. 8*d*. to the fraction of a pound.

Here 1*s*. 8*d*. = 20 pence. ⎫ ∵ the fraction is $\frac{20}{240} = \frac{1}{12}$
and £1 = 240 pence. ⎭ or 1*s*. 8*d*. is the $\frac{1}{12}$ of a £.

**Rule 3.** To reduce a fraction of one quantity to a fraction of another. Express the first quantity as a fraction of the latter, and multiply the two fractions together.

Ex. Reduce $\frac{2}{3}$ of £1 to the fraction of a guinea.

Here one pound contains 20*s*., and a guinea 21*s*.; ∵ a pound is $\frac{20}{21}$ of a guinea, and $\frac{2}{3}$ of a pound is $\frac{2}{3}$ of $\frac{20}{21}$ or $\frac{40}{63}$ of a guinea.

---

will be the same when we have reduced to the same denomination, as reducing does not alter the value of any quantity. But as the numbers now mean the same thing, the first will be the same fraction as the second, as the first number is of the second; and if we divide unity into the latter number of parts, and take the first number of them, it is evident this will express the fraction one is of the other; thus in the Ex. 1*s*. 8*d*. = 20 pence, £1 = 240 pence. Hence 1*d*. is the $\frac{1}{240}$th of a pound, ∵ 20 pennies are $\frac{20}{240}$ or $\frac{1}{12}$ of £1.

3. This rule comes under multiplication of fractions. Instead of a fraction of one quantity we must take a fraction of that

## ADDITION OF FRACTIONS OF CONCRETE QUANTITIES.

47. Rule 1. To express several fractions as one fraction, in any required denomination. Reduce all the fractions to other fractions of the required denomination: reduce to a common denominator, and add the numerators.

Ex. Reduce $\frac{2}{3}$ of a shilling $\frac{4}{5}$ of a penny to the fraction of a pound.

$$\left.\begin{aligned}\frac{2}{3}\text{ o a shilling } &= \frac{2}{3\times 20} = \frac{1}{30}\text{ of a pound}\\ \frac{4}{5\times 12\times 20} &= \frac{1}{300}\text{ of a pound}\end{aligned}\right\}\begin{aligned}&\text{and }\frac{1}{30}+\frac{1}{300} =\\ &\frac{11}{300}\text{ of a £.}\end{aligned}$$

Rule 2. When it is required to express the fractions in the denominations themselves. Find the value of each fraction (by 46), and add the separate values as required.

Ex. Add $\frac{3}{4}$ of a pound, $\frac{5}{6}$ of five shillings, and $\frac{2}{3}$ of a shilling.

£$\frac{3}{4}$ = 15*s*., $\frac{5}{6}$ of five shillings = 4*s*. 2*d*., $\frac{2}{3}$ of a shilling = 8*d*.

| *s*. | *d*. |
|---|---|
| 15 | 0 |
| 4 | 2 |
| | 8 |
| 19*s*. | 10*d*. |

---

quantity expressed in another denomination, that is, generally, a fraction of a fraction. Thus, in the Ex. we have $\frac{2}{3}$ of a pound to reduce to a fraction of a guinea. Now one pound is $\frac{20}{21}$ of a guinea; $\frac{2}{3}$ of one pound is, therefore, $\frac{2}{3}$ of $\frac{20}{21}$ of a guinea. Or the rule may be explained thus:—$\frac{2}{3}$ of a pound is $\frac{2}{3}$ of twenty shillings, or, $\frac{2\times 20}{3}$ shillings. Now, as there are twenty-one shillings in a guinea, to express this that it may mean parts of a guinea, we must evidently divide by 21, as now the unit is 21 times as large; we thus get $\frac{2\times 20}{3\times 21}$ or $\frac{40}{63}$

48. In Subtraction we proceed in the same manner, except that we must take the difference at last.

49. Multiplication of concrete quantities, or fractions of them, by fractional multipliers.

RULE. Reduce the quantity to the lowest denomination, express it as a fraction of the highest; reduce the multiplier to a fraction (if necessary), multiply the two fractions together, and express the result in the proper denominations.

Ex. Multiply 2 months, 3 weeks, and 6 days, by $3\frac{1}{3}$.

Here 2 mo., 3 w., and 6 d., $= 60 + 21 + 6 = 87$ days $= \frac{87}{30}$ of a month, and $\frac{87}{30} \times \frac{10}{3} = \frac{29}{3}$ of a month $=$ 9 months and 20 days.

This rule is similar to 46, Rule 1.

50. Division of concrete quantities, or fractions of them, by fractional divisors.

RULE. Proceed as before, but invert the divisor.

Ex. Divide £1 5*s.* 6*d.* by $1\frac{1}{5}$.

Here £1 5*s.* 6*d.* $=$ 51 sixpences; and there are 40 sixpences in a pound, ∴ the fraction is $\frac{51}{40}$; and $1\frac{1}{5} = \frac{6}{5}$.

$$\frac{51}{40} \div \frac{6}{5} = \frac{51}{40} \times \frac{5}{6} = \frac{51}{48} = £1\ 1s.\ 3d.$$

51. Simple Proportion in Fractions.

Here we must state, as in the ordinary rule. Invert the first term, and multiply all the terms together, taking care to strike out factors.

This is because we have to divide in the ordinary rule by the first term; but to divide by a fraction we must invert it, and proceed as in multiplication.

Ex. If $\frac{4}{3}$ yard cost $\frac{7}{6}$ of a shilling, what will $\frac{8}{5}$ yard cost?

$$\begin{array}{ccccccc} \text{Yard.} & & \text{Yard.} & & \text{Shilling.} & & \qquad\qquad\qquad\qquad\qquad s.\ \ d. \\ \frac{4}{3} & : & \frac{8}{5} & :: & \frac{7}{6} & & = \frac{3}{4} \times \frac{8}{5} \times \frac{7}{6} = \frac{7}{5} = 1\ \ 4\frac{3}{4}\ \frac{1}{5}. \end{array}$$

---

(49—52.) There is nothing new in principle in these rules.

Multiplication and division may sometimes be more easily performed by Practice.

## QUESTIONS.

A 1. What is a Fraction? 2. Give the meaning of the word. 3. How is it written? 4. What is the Numerator? 5. What the Denominator? 6. Is there any other definition of a fraction? 7. What does $\frac{2}{8}$ mean? 8. What $\frac{2}{3}$ of 4? 9. What is $\frac{3}{3}$ of 4? 10. Explain Proper Fraction. 11. Improper Fraction. 12. Mixed Numbers; giving examples. 13. What is a Simple Fraction? 14. A Compound Fraction? 15. A Complex Fraction? Give instances.

B 1. How must fractions be reduced to their lowest terms? 2. How may whole numbers be expressed as as fractions? 3. How must a fraction be reduced to another having a given denominator? 4. How must fractions be reduced to others having a common denominator?

C 1. Shew how to reduce improper fractions to mixed numbers. 2. Mixed numbers to improper fractions. 3. Complex to simple fractions.

D 1. How must fractions be added together? 2. How must fractions be subtracted from fractions? 3. To multiply a fraction by a whole number, must we multiply the numerator, or denominator? 4. How must we divide by a whole number? 5. How must fractions be multiplied by fractions? 6. How must fractions of fractions be found? 7. How must fractions be divided by fractions?

E. 1. Shew how to find the value of a fraction of a given quantity. 2. Shew how to express a quantity as a fraction of another. 3. Shew how to reduce a fraction of one quantity to a fraction of another.

F 1. How must fractions of concrete quantities be added together, and be expressed as one fraction? 2. How in proper denominations? 3. How must concrete quantities, or fractions of them, be multiplied by fractional multiplier? 4. How be divided?

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## DECIMAL FRACTIONS.

52. By a decimal we mean a quantity always less than 1, but which may consist of several figures, as 75; and to shew that we do not mean the number 75, but a quantity less than one, we place a point before it, and write it ·75.

A decimal is said to have as many decimal places as it has figures after the decimal point.

RULE AND DEFINITION. Every decimal may be represented as a proper fraction, having for its numerator the figures of the decimal without the point, and a denominator 1, followed by as many ciphers as there are decimal

---

(52.) A decimal is only another way of writing a fraction, having the figures of the decimal as its numerator and denominator, followed by as many ciphers as there are decimal places; thus, ·2 and $\frac{2}{10}$, ·25 and $\frac{25}{100}$, or $\frac{20}{100}$ and $\frac{5}{100}$, that is, $\frac{2}{10}$ and $\frac{5}{100}$; similarly ·253 and $\frac{253}{1000}$, or $\frac{2}{10}+\frac{5}{100}+\frac{3}{1000}$, &c., are respectively identical. Hence it appears that the figures decrease in a tenfold proportion as we go further from the decimal point, and that the figure standing next to it is $\frac{1}{10}$ of the same before it; and that standing in any place, is equal to a fraction whose numerator is the number, and the denominator ten multiplied together as many times as there are decimal places (counting the figure itself) between it and the decimal point. So that we may write a decimal and a whole number together, as 527·253, and every figure increases in value tenfold over the same figure one place nearer the right hand.

places, and must always, when so required to be represented, be reduced to its lowest terms.

Thus, $\cdot 7$ is the same as $\frac{7}{10}$; $\cdot 75$ as $\frac{75}{100}$, or $\frac{3}{4}$.

If there are ciphers just after the decimal point, they must be counted, thus, $\cdot 075$ is not $\frac{75}{100}$, but $\frac{075}{1000}$, that is, $\frac{75}{1000}$, or $\frac{3}{40}$.

When a whole number is written before a decimal, it means that the decimal is to be added to the whole number, and together, they are equivalent to an improper fraction, or mixed number; thus, $5\cdot 376$ means $\cdot 376$ added to 5, and is equal to $5\frac{376}{1000}$, or $5\frac{47}{125}$.

Ciphers added to the end of a decimal make no difference, $\cdot 75$, $\cdot 750$, $\cdot 75000$, or $\frac{75}{100}$, $\frac{750}{1000}$, $\frac{7500}{10000}$, are all the same.

53. To express a fraction as a decimal.

**Rule.** Add as many zeros as are necessary to the numerator; divide by the denominator, and mark off as many figures from the end of the quotient as we have added ciphers to the numerator.

Ex. Express $\frac{7}{8}$ as a decimal.

$$\begin{array}{r} 8\,)\,7\cdot 000 \\ \hline \cdot 875 \end{array}$$

$\cdot 875$ is the decimal, as there are 3 ciphers added.

Express $\frac{33}{4}$ as a decimal.

$$\begin{array}{r} 4\,)\,33\cdot 00 \\ \hline 8\cdot 25 \end{array}$$

$8\cdot 25$ is the answer, as we have added 2 ciphers.

---

(53.) $\frac{7}{8} = \frac{7\times 1000}{8\times 1000} = \frac{1}{1000} \times \frac{7000}{8} = \frac{1}{1000} \times 875 = \cdot 875$

The reason of this rule is, that in affixing these ciphers to the numerator, we really multiply it by 10 or 100, &c., as the case may be, so the quotient will be 10 or 100 &c. times too large; but if we mark off just as many figures as there are decimal places, we, in reality, divide the quotient by 10 or 100, &c., and thus get the true quotient.

## ADDITION OF DECIMALS.

54. Rule. Place so that the decimal points may be in the same line; add as in whole numbers, and put the decimal point in the same line.

Ex. Add 7·58, ·364, 2·0547 together.

$$\begin{array}{r} 7{\cdot}85\phantom{00} \\ {\cdot}364\phantom{0} \\ 2{\cdot}0547 \\ \hline 10{\cdot}2687 \end{array}$$

---

N.B. Instead of writing "1 followed by a certain number of ciphers," we sometimes say, "10 raised to that power," *i.e.*, 10 multiplied by itself that number of times.

Thus, 100, 10 × 10, 2nd power of 10 squared; 1000, 10 × 10 × 10, 3rd power of 10 cubed; 10000, 10 × 10 × 10 × 10, 4th power of 10, &c., all respectively denote the same numbers. Again, as each time we multiply by 10, another cipher is added to the number, and if by ten twice, thrice, &c., two, three, &c., ciphers are added. Hence it follows in multiplying two numbers, each being 1 multiplied by a different number of ciphers, the product will be 1, followed by as many ciphers as there are in both together, *i.e.*, 10 raised to one power multiplied, 10 raised to another power is equal to 10 raised to sum of the powers: similarly, dividing by 10, 100, &c., is the same as striking off one, two, &c., ciphers.

(54.) Since

$$\begin{array}{rl} 7{\cdot}85 = & 7 + \frac{8}{10} + \frac{5}{100} \\ {\cdot}364 = & \phantom{7 +} \frac{3}{10} + \frac{6}{100} + \frac{4}{1000} \\ 2{\cdot}0547 = & 2 \phantom{+ \frac{8}{10}} + \frac{5}{100} + \frac{4}{1000} + \frac{7}{10000} \\ \hline & 9 + \frac{11}{10} + \frac{16}{100} + \frac{8}{1000} + \frac{7}{10000} \\ & 10 + \frac{2}{10} + \frac{6}{100} + \frac{8}{1000} + \frac{7}{10000}, \text{ or } 10{\cdot}2687. \end{array}$$

The reason of this rule is easily seen: converting the decimal into fractions, each figure divided by the proper power of ten, it is evident that we can only add or subtract from each other those numerators which have the same power of ten as denominators, that is, are at the same distance from the decimal point.

## SUBTRACTION OF DECIMALS.

55. **Rule.** Place as in Addition; subtract as in whole numbers, and put the decimal point under the others.

Ex. Subtract ·37 from 4·16.

$$\begin{array}{r} 4{\cdot}16 \\ {\cdot}37 \\ \hline 3{\cdot}79 \end{array} \qquad \begin{array}{rl} 4{\cdot}16 \text{ is} & 4 + \tfrac{1}{10} + \tfrac{6}{100} \\ {\cdot}37 \text{ is} & \tfrac{3}{10} + \tfrac{7}{100} \\ \hline \text{Subtracted} & 3 + \tfrac{7}{10} + \tfrac{9}{100}, \text{ or } 3{\cdot}79. \end{array}$$

## MULTIPLICATION OF DECIMALS.

56. **Rule.** Multiply as in whole numbers, and mark off as many decimal places in the product as there are in the multiplier and multiplicand together.

Ex. Multiply 3·57 by ·421.

$$\begin{array}{r} 3{\cdot}57 \\ {\cdot}421 \\ \hline 357 \\ 714\phantom{0} \\ 1428\phantom{00} \\ \hline 1{\cdot}50297 \end{array}$$

Marking off 5 decimal places, as there are 3 in the multiplier and 2 in the multiplicand.

To multiply a decimal by a fraction, we must either reduce the latter to a decimal, and multiply as in this rule; or, multiply by the numerator, and divide the product by the denominator, by the rule given in Art. 57.

---

(56.) $3{\cdot}57 \times {\cdot}421 = \frac{357}{100} \times \frac{421}{1000} = \frac{357 \times 421}{100 \times 1000} = \frac{150297}{100000}$ or $1{\cdot}50297$.

Expressing the multiplier as a fraction, and the multiplicand as another, and multiplying, as in fractions, the numerators together, also the denominators, the new denominator will manifestly contain as many factors of ten as there are in the multiplier and multiplicand together.—(See remarks end of 53.)

That is, the product will have as many decimal places as there are in them together, since each factor, ten, is equivalent to one decimal place.

## DIVISION OF DECIMALS.

57. **Rule.** Divide as in whole numbers; if the decimal places be equal there will be none in the quotient; if unequal, mark off in the quotient the number of decimal places the dividend has more than the divisor; or add as many ciphers at the right hand as it has less.

If ciphers be added to the dividend, these must be counted in the number of decimal places.

In practice, the decimal point should be put, or at least fixed in the mind, before dividing.

Ex. 1. Divide 45·75 by 25

```
·25 ) 45·75 ( 183
      25
      ---
      207
      200
      ---
        75
        75
        --
```

As the decimal places are equal.

Ex. 2. ·36 by ·25

```
·25) ·3600 ( 1·44
      25
      ---
      110
      100
      ---
       100
       100
       ---
```

As there are 2 more in the diyidend.

Ex. 3. 457·5 by ·025

```
·025 ) 457·5 ( 18300·
       25
       ---
       207
       200
       ---
        75
        75
        --
```

As there are 2 more in the divisor.

---

(57.) Ex. 1. $45·75 \div ·25 = \frac{4575}{100} \div \frac{25}{100} = \frac{4575}{100} \times \frac{100}{25} = \frac{4575}{25} \times \frac{100}{100} = 183.$

Ex. 2. $·36 \div ·25 = ·3600 \div ·25 = \frac{3600}{10000} \div \frac{25}{100} = \frac{3600}{25} \times \frac{100}{10000} = 144 \times \frac{1}{100} = 1·44.$

Ex. 3. $457·5 \div ·025 = 457\frac{5}{10} \div \frac{25}{1000} = \frac{4575}{25} \times \frac{1000}{10} = 183 \times 100 =$ 18300.

Expressing as fractions, inverting the divisor (44), and then multiplying, we shall have the numerator of the dividend multiplied by as many factors of ten as there are decimal places in the divisor (or denominator); the divisor multiplied by as many factors of ten as there are decimal places in the dividend.

We may therefore strike from the numerator and denominator the same number of factors, and the result will be, we shall have to multiply the result of the division of the whole numbers by the excess of factors of ten in the divisor over those in the dividend; that

## CIRCULATING DECIMALS.

58. Sometimes we find decimals whose figures go on perpetually recurring, as ·568468468468 46... These are called *Circulating Decimals*. The part recurring is called the period, and is generally written only once, with a

---

is, affix as many ciphers to it as the number of that excess, write the quotient as a whole number, if they be equal, or divide by the excess of those in the dividend over the divisor, and therefore strike off as many decimal places.

(58.) If we have a fraction reduced to its lowest terms to express as a decimal (the whole number being first excluded, if the fraction be improper), we must add ciphers before we divide, and place them at the end of each remainder; therefore these will all terminate in one of the numbers, 10, 20, 30 .... 90. Now, if the last figure of the denominator be 1, 3, 7, or 9, this can never be multiplied by a number less than 10, so as to give a product ending in 0. Hence there will always be a remainder, and it is clear that as we have to divide continually, when we arrive at a remainder the same as one before, we shall get the same quotient, and therefore the same second remainder, or all, will recur; that is, we shall have a circulating decimal.

Again; If the denominator should not end in 1, 3, 7, or 9, but we can multiply the numerator and denominator by ten any number of times, and (not considering the tens in the denominator) can strike out factors in the first denominator, and the tens of the numerator, so as to make the former end in 1, 3, 7, or 9, and then have no common factor with what is left in the numerator, we shall also, in this case, have a circulating decimal, because the tens in the denominator, which we must divide by afterwards, will only affect the situation of the decimal point. But, as we observe that any number ending in 1, 3, 7, or 9 is not divisible by 2 or 5, and these (2 and 5) are the only numbers which will divide 10, or can be excluded by this operation, we conclude that if the denominator of a fraction reduced to its lowest terms, contain other

dot over the first figure, as in the Example, 846 is the period, and the decimal is written $\cdot 56\dot{8}46$.

To express circulating decimals as fractions.

RULE 1. Where all the figures circulate. Write as numerator the circulating period, and denominator as many 9's as there are figures that circulate.

Ex. Reduce ·3636...to a fraction.

Here the figures in the period are 36, and two in number; therefore the fraction is $\frac{36}{99}$ or $\frac{4}{11}$.

RULE 2. When all do not circulate. Write as numerator the figures between the decimal point and the end of the first period, subtracting the figures which do not circulate; and as denominator as many 9's as there are

---

factors than 2 or 5, that is, after excluding these (if necessary), it be greater than 1, and ending in 1, 3, 7, or 9, it will always produce a circulating decimal.

The number of figures in the period, at most, will be always one less than the denominator; for, writing down all the different numbers between zero and the denominator, we observe they are in number *one less*; hence, as every remainder is always less than the divisor (or denominator), if we consider *as many* remainders as there are units in the denominator, the last must necessarily be the same as one before; it will therefore give the same quotient and remainder as the other, or the figures will recur and *may* have recurred from the first.

To prove the examples, 100 × 3636... = 36·3636... = 36 + ·3636... Now the decimal part on the right-hand is the same as before, as the figures 36 are repeated for ever, therefore subtracting ·3636... from each side we get 99·3636... = 36; therefore ·3636... = $\frac{36}{99}$.

Again: 100·000 × $\cdot 56\dot{8}46\ldots = 56846\cdot\dot{8}46\ldots = 56846 + \cdot\dot{8}46\ldots$

And 100 × $\cdot 56\dot{8}46\ldots = \cdot 56\cdot\dot{8}46\ldots = 56 + \cdot\dot{8}46\ldots$

Now the decimal part $\dot{8}46$... obtained in the first operation is the same as that in the second, since 846 is repeated for ever; but if equals be taken from equals the remainders are equal, therefore

figures in the period, followed by as many zeros as there are figures which do not circulate.

Ex. Reduce $\cdot 56\dot{8}4\dot{6}$ to a fraction.

Here 846 is the period, and three in number; 56 the figures which do not circulate; therefore the fraction will be

$$\frac{56846-56}{99{,}900} = \frac{56790}{99{,}900} = \frac{631}{1110}$$

Circulating decimals should be expressed as fractions before any operation is performed upon them, or the circulating part extended as far as required, and the rest neglected.

---

subtracting the right-hand side of the latter from that of the former, and the left-hand side from the left, we obtain

$$\begin{array}{lcl} 99{,}900 \times \cdot 56\dot{8}46\ldots\ldots & = & 56846 - 36 \\ \therefore \quad \cdot 56\dot{8}46\ldots\ldots & = & \dfrac{56846 - 56}{99{,}900} \\ & = \dfrac{56790}{99{,}900} & = \dfrac{631}{1110} \end{array}$$

To prove the second Rule (of which the first is a particular case) we observe that if we subtract one from unity followed by any number of ciphers, the result would be as many nines as there are ciphers.

Hence, multiplying by any number of tens, or adding the same number of ciphers to each, we observe the result would then have as many nines as there are ciphers in the larger number, followed by as many ciphers as there are in the less, and, therefore, if each had been multiplied by any other number, their difference would be the number multiplied by the figures so determined. Now, multiplying a circulating decimal by 1, followed by as many ciphers as there are figures to the end of the first period, the result would be a whole number to the end of the first period, followed by the circulating part as decimal. Similarly, multiplying by 1, followed by as many ciphers as there are figures which do not circulate, we should obtain a whole number to the commencement of the first period, followed by the circulating part as decimal.

Hence, subtracting equals from equals, or the decimal we have

## REDUCTION OF DECIMALS.

59. RULE 1. To express a decimal of one denomination by means of lower denominations. Multiply the decimal by the number in the next lower denomination; mark off the whole number, if any. Multiply the decimal part left by the number in the next lower denomination, and continue to the lowest denomination required. The numbers marked off will be in the successive lower denominations.

Ex. Find the value of ·347 of a £.

```
  ·347
    20
 ------
 6·940 shillings...6 is not multiplied because it is a whole
    12                       number of shillings.
 ------
11·280 pence.  6s. 11¼d. ·120  ANSWER.
     4
 ------
 1·120 farthings.
```

RULE 2. To reduce any quantity to the decimal of any other. Reduce to a common denomination, and divide

---

multiplied last from the other, with each multiplier preserved, and the last result from the first, we find that the decimal multiplied by as many nines as there are figures which circulate, with as many ciphers attached as there are figures which do not, is equal to the difference of the whole numbers thus formed, since the decimal parts being equal will destroy each other; hence, dividing both sides by the multiplier, we obtain the same result as by the rule.

(59.) A decimal representing tenths of any whole; the 1st Rule is derived because to reduce any quantity to a lower denomination we multiply by the number of that lower denomination in the first. (Vide 46th Rule.) We mark off by the rules proved in multiplication.

The next rule is 46, 2 carried out a step further, namely, to

the number in the given quantity by that in the denomination demanded.

Ex. 1. Reduce 4*s*. to the decimal of a pound; also 1*s*. 3*d*.

Ex. 1. 20)4·0

·2

Ex. 1. Since there are 20*s*. in £1.

Ex. 2. 240)15000(·625
1440

600
480

1200
1200

Ex. 2. Since 1*s*. 3*d*. = 15*d*., and £1 = 240*d*.

It is more convenient when there are several denominations, to reduce the lowest to the decimal of the next higher, add to it the part in that denomination, and reduce again.

Ex. Reduce 17*s*. 3¾*d*. to the decimal of £1; also of £5.

Qr.
4)3·00

12)3·7500 Because 3¾*d*. is 3.75 pence.

20)17·3125 Because 17*s*. 3¾*d*. is 17.3125 shillings.

5)·865625 of £1 As decimal of £1 must be ⅕ of the

·173125 of £5 decimal of £5.

Rule 3. To express a decimal of one quantity as a decimal of another. Express the first quantity as a fraction of the second; multiply the given decimal by the numerator, and divide by the denominator; or reduce the fraction to a decimal before multiplying.

Ex. Reduce ·78 of a penny to the decimal of a shilling.
Here one penny is $\frac{1}{12}$ of a shilling.

12)·78

·065 of a shilling.

---

actual division: thus 4*s*. expressed as the fraction of £1 is $\frac{4}{20}$, or ·2. So also Rule 3 is (46, 3) similarly extended.

The first rule is the most important, the others are rarely of use.

Ex. Express ·45 of 2*s*. 1*d*. as the decimal of 5*s*. 6*d*.

```
66)11·25(·17045        ·45
   66                  25
   ---                ----
   465                225
   462                 90
   ---                ----
    300              11·25
    264
    ---             Since 2s. 1d. is 25d.,
     360            5s. 6d. is 66d.
     330
     ---
      30  ∴ ·45 of 2s. 1d. is ·17045... of 5s. 6d.
```

**60.** To add or subtract decimals of different denominations (but of the same kind) we may first express all as decimals of some one denomination; or find the value of each in units of lower denominations.

Simple Proportion of Decimals merely consists in reducing, if necessary, the quantities to decimals before multiplying and dividing.

---

61. Sometimes in Multiplication or Division of Decimals, it is only required to get the result true to a certain number of decimal places; and it is therefore not necessary to multiply all the figures in the multiplicand by every one in the multiplier, or to divide by every one in the divisor.

---

## CONTRACTED MULTIPLICATION.

Rule. Place the units figure (or if there be more, add ciphers, and place the cipher in the units place in the multiplier) under the last decimal place required to be kept in the product; reverse the multiplier (that is, place the lowest decimal place highest), keeping the

---

(61.) The figure in the units place would cause no change in the situation of the decimal point in the product.

Hence, to get the result true to any required number of places, we need only multiply the figure next lower than the required decimal place by it, and, even of that sum, only write down the part which would affect the next figure.

units place as before; multiply by the figure which stands nearest the right hand, first the figure in the multiplicand next lower; carry to add up with the next product the number of tens (or that next greater, if the last figure of the product exceed four), and neglect the last figure; then multiply all the figures which stand higher, as in common multiplication: and so proceed, always multiplying first the figure next lower, neglecting the units, carrying the tens as before, and placing the first figures beneath each other; add up all the rows, and mark off the decimal places required.

The figures which have none standing above them may be omitted after the first.

Ex. Multiply 5·624732 by 24·16375 and get the result true to 3 decimal places.

| | |
|---|---|
| 5·624732 | |
| 57361·42 | |
| 112495 | ∵ 2 × 3 is 6, we add 1. |
| 22499 | 4 × 7 is 28, we add 3. |
| 562 | 1 × 4 is 4, we have |
| 337 | nothing to add. |
| 17 | |
| 4 | |
| 135·914 | |

| | | |
|---|---|---|
| | 5·624732 | |
| | 24·16375 | |
| | 28123660 | Multiplying |
| 3 | 9373124 | as in the |
| 16 | 874196 | common |
| 337 | 48392 | rule. |
| 562 | 4732 | |
| 22498 | 928 | |
| 112494 | 64 | |
| 135·914 | 61786500 | |

When perfect accuracy is of importance, it is often necessary to calculate for one place more than required.

## CONTRACTED DIVISION.

62. When only a certain number of decimal places are wanted in the quotient, we must use the following rule:—

---

Now, multiplying by a decimal makes every place lower, therefore, in multiplying by the first decimal digit, we need not consider so many, but one fewer: similarly for each successive decimal place, standing further from the decimal point, we need consider one fewer.

(62.) The explanation of this is exactly the same as the preceding; as the figures of the quotient are more to the right hand they

Make the dividend (if necessary) not less than the divisor; divide as usual, but consider the remainder only, and cut off in the divisor the right hand figure, or more, so as to make it less than the remainder; divide again (adding the tens, or the next number when the figure cut off would have given a number whose last figure is greater than 4), and so continue, always making the divisor less than the remainder.

Ex. Divide 5·7368 by ·346 to three places.

| | | | | |
|---|---|---|---|---|
| | 3·46527 ) 573670 ( 1·6554 | 3·46527 ) 573680 ( 1·6554 | | |
| | 346527 | 346527 | | |
| | 227143 | 227143 | 0 | |
| | 207916 | 207916 | 2 | |
| It is better to put a mark under each figure in the divisor when it is struck out. | 19227 | 19226 | 80 | Dividing as in the common |
| | 17326 | 17326 | 35 | |
| | 1901 | 1900 | 450 | |
| | 1732 | 1732 | 635 | |
| | 169 | 167 | 8150 | |
| | 138 | 138 | 6108 | |
| | 31 | 29 | 2042 | |

It is generally better to find the result true to one place more than required.

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## QUESTIONS.

A 1. What is a Decimal? 2. How is it written, and why? 3. How many decimal places is a decimal said to have? 4. How may a decimal be represented as a fraction? 5. What fraction is ·7 equal to? What ·75? 6. Do ciphers make any difference just after the decimal point? 7. Do they make any difference at the end of the

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become of less value, and one multiplied into the same figure as that next before the product, would be in the decimal place next lower than the former product.

decimal? 8. What does a whole number written before a decimal denote? 9. What are they together equal to?

B 1. How must a fraction be expressed as a decimal? 2. How must decimals be added together? 3. Where must the decimal point be placed after adding together? 4. How must decimals be subtracted from decimals? 5. How must decimals be multiplied by decimals? How many decimal places will there be in the product? 7. How must decimals be divided by decimals? 8. Must the ciphers added to the dividend be counted? 9. When the dividend has as many decimal places as the divisor, how many will there be in the quotient? 10. How many when the divisor has less? 11. How many when the divisor has more?

C 1. What are circulating decimals? 2. What is the period? 3. How are circulating decimals generally written? 4. How must they be expressed as fractions when all the figures recur? 5. How when only a part?

D 1. How must a decimal be expressed in lower denominations? 2. How must any quantity be expressed as the decimal of any other? 3. How when there are several denominations? 4. How must a decimal of any quantity be expressed as the decimal of any other?

# CHAPTER VI.

## PRACTICE.

Practice contains rules for finding the price of any number of things at a certain price each.

The number is written down at the rate of one in a proper denomination, and such multipliers and parts are taken of this denomination that, added together, they will be equal to the given rate; the same parts, &c., of the number written down will give the required price.

CASE 1. To take parts of a penny. Write the number to be bought as pence, take a part of this not greater than the part required the price is of a penny; the remainder will be in pence, and must be reduced to farthings; take next a part of this part, and continue till we have taken as many parts as will make up the given price. Add all the quotients and remainders together; the sum of the former will be in pence, and must be brought to shillings and pounds.

Ex. Find the price of 347 yards at $\frac{1}{6}d.$ per yard.

| | | |
|---|---|---|
| $d.$ | | |
| $\frac{1}{6} = \frac{1}{6}$ | 347 | Since the price at a penny is 6 times too much, we therefore take a sixth, that is divide it by 6. |
| 12) | $57\frac{3}{4}$ | We take the farthing next less than $\frac{5}{6}$. |
| | $4s.\ 9\frac{3}{4}d.$ | |

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The number to be bought is kept the same to keep the attention fixed on the difference caused by the changes in the price. In the 2nd example, Case 1, there is half of a farthing in the second part, which is omitted; fractions of farthings are left out in all the examples.

Ex. Find the price of 347 at $\frac{3}{8}d.$

| $d.$ | | |
|---|---|---|
| $\frac{1}{4} = \frac{1}{4}$ | 347 | |
| $\frac{1}{8} = \frac{1}{2}$ | $68\frac{3}{4}$ | |
| | $43\frac{1}{4}$ | omitting the half farthing. |
| $\frac{3}{8}$ | 12) 130 | |
| | $10s.\ 10d.$ | |

Here $\frac{3}{8} = \frac{2+1}{8} = \frac{2}{8} + \frac{1}{8} = \frac{1}{4} + \frac{1}{8}$

The price at $\frac{1}{4}d.$ is the quarter of that of a $1d.$, and $\frac{1}{8}$ the half of that at $\frac{1}{4}d.$

CASE 2. When the price is less than a shilling.

Ex. Find the price of 347 at $2\frac{7}{8}d.$

| $d.$ | | |
|---|---|---|
| | | 347 |
| 2 | | 2 |
| $\frac{1}{2} = \frac{1}{2}$ | | 694 |
| $\frac{1}{4} = \frac{1}{2}$ | | $173\frac{1}{2}$ |
| $\frac{1}{8} = \frac{1}{2}$ | | $86\frac{3}{4}$ |
| | | $43\frac{1}{4}$ |
| $2\frac{7}{8}$ | 12) | $997\frac{1}{2}$ |
| | 2,0) | 8,3 1 |
| | | £4 3 $1\frac{1}{2}$ |

Here $\frac{7}{8} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$.

The price at $2d.$ is twice that of a $1d.$, at $\frac{1}{2}d.$ half that at a $1d.$, at $\frac{1}{4}d.$ half that at a $\frac{1}{2}d.$, and at $\frac{1}{8}d.$ half that at $\frac{1}{4}d.$

CASE 3. To take parts of shillings. The same as in Case 1, except that now the number written is shillings, therefore the remainder and the quotient are shillings.

Ex. Find the price of 347 at $5\frac{1}{8}d.$

| $d.$ | | | |
|---|---|---|---|
| $4 = \frac{1}{3}$ | | 347 | |
| $1 = \frac{1}{4}$ | | 115 | 8 |
| $\frac{1}{8} = \frac{1}{8}$ | | 28 | 11 |
| | | 3 | $7\frac{1}{4}$ |
| $5\frac{1}{8}$ | 2,0) | 14,8 | $2\frac{1}{4}$ |
| | | £7 8 | $2\frac{1}{4}$ |

Because the price at $4d.$ is one third that of a shilling; at $1d.$ one quarter that of $4d.$, &c.

When the price is an exact part of $1s.$, as $1d.$, $1\frac{1}{2}d.$, &c., we must take that part of the upper line.

**Case 4.** When the price is more than a shilling.

Ex. 347 at 3*s.* 4⅞*d.*

| *s.* | *d.* | | | | |
|---|---|---|---|---|---|
| | | | | 347 | |
| 3 | | | | 3 | |
| | | | | 1041 | |
| | 4 | = | ⅓ | 115 | 8 |
| | ½ | = | ⅛ | 14 | 5½ |
| | ¼ | = | ½ | 7 | 2¾ |
| | ⅛ | = | ½ | 3 | 7¼ |
| 3 | 4⅞ | | 2,0) | 118,1 | 11½ |
| | | | | £59 1 | 11½ |

Because the price at 3 shillings is 3 times the price at one shilling, or 3 times 347 shillings: at 4*d.* is ⅓ that at one shilling, &c.

**Case 5.** To take parts of a pound. The same as in Case 1, except that the number written is pounds, and remainder must be brought to shillings and pence.

Ex. 347 at 16*s.* 5⅛*d.*

| *s.* | *d.* | | | | | |
|---|---|---|---|---|---|---|
| | | | | 347 | | |
| 10 | | = | ½ | 173 | 10 | |
| 5 | | = | ½ | 86 | 15 | |
| 1 | | = | ⅕ | 17 | 7 | |
| | 4 | = | ⅓ | 5 | 15 | 8 |
| | 1 | = | ¼ | 1 | 8 | 11 |
| | ⅛ | = | ⅛ | | 3 | 7¼ |
| 16 | 5⅛ | | | £285 | 0 | 2¼ |

Because the price at 10*s.* is ½ that at one pound, or of 347 pounds; at 5*s.* ½ that at 10*s.*, &c.

If the price be an exact part, as 3*s.* 4*d.* = £⅙, or 6*s.* 8*d.* = £⅓, we must take ⅙ or ⅓ off at once.

**Case 6.** When the price is more than a pound.

Ex. 347 at £3 6*s.* 4⅝*d.*

| £ | *s.* | *d.* | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | | 347 | | |
| 3 | | | | | 3 | | |
| | | | | | 1041 | | |
| | 5 | | = | ¼ | 86 | 15 | |
| | 1 | | = | ⅕ | 17 | 7 | |
| | | 4 | = | ⅓ | 5 | 15 | 8 |
| £3 | 6 | 4 | | | £1150 | 17 | 8 |

Because the price at £3 is 3 times that of £1, or 3 × £347; at 5*s.*, one quarter that at a pound, so we take a quarter of 347, &c.

These six cases come under the following general rules.

Cases 1, 3, 5. Rule 1. Write the price down at the rate of a unit of the next higher denomination, and place a line below it. Find a part of this denomination, not greater than the given price, take the same part of the upper line and place it below, the remainder will be in the denomination of the top line, and must be reduced lower and divided again. This will be the price at that rate. Next find a part of this rate, take the same part of the last quotient, and continue till we have taken as many parts as make up the required rate. Add all the rows together, bringing the sum to higher denominations if possible.

Cases 2, 4, 6. Rule 2. Multiply the number of things by the number in the highest denomination; take parts of a unit of the highest denomination, the same part of the top line, place it below the quantity just found; take parts again till we have taken as many parts as make up, with the multiplier, the given price.

N.B.—The figures standing opposite each part will be the price at that rate.

Case 7. Sometimes the price is such that we cannot take the successive parts of parts of each rate which will make up the given price. It is then better to reduce the two first denominations to that of the lower denomination, or continue taking parts of the upper line, or of any line which may be convenient; take parts not exact shillings, &c.

The methods which have preceded should, however, always be adopted when practicable, as they are the most direct and involve the least risk of error.

Ex. 347 at £3 17s. 11¼d.

| £ s. d. | | 347 | | |
|---|---|---|---|---|
| 3 17 | | 77 | | |
| | | 2429 | | |
| | | 2429 | | |
| | | 26719 | | |
| 6 | = ½ | 174 | 6 | |
| 4 | = ⅓ | 115 | 8 | |
| 1 | = ¼ | 28 | 11 | |
| ¼ | = ¼ | 7 | 2¾ | |
| £3 17 11¼ | 2,0) | 2704,4 | 3¾ | |
| | | £1352 | 4 | 3¾ |

| £ s. d. | | 347 | | |
|---|---|---|---|---|
| 3 | | 3 | | |
| | | 1041 | | |
| 10 | = ½ | 173 | 10 | |
| 4 | = ⅕ | 69 | 8 | |
| 2 | = ½ | 34 | 14 | |
| 1 | = ½ | 17 | 7 | |
| 6 | = ½ | 8 | 13 | 6 |
| 4 | = ⅓ | 5 | 15 | 8 |
| 1 | = ¼ | 1 | 8 | 11 |
| ¼ | = ¼ | | 7 | 2¾ |
| £3 17 11¼ | | £1352 | 4 | 3¾ |

Ex. 347 at £3 18s. 4d.

| £ s. d. | | 347 | | |
|---|---|---|---|---|
| 3 | | 3 | | |
| | | 1041 | | |
| 10 0 | = ½ | 173 | 10 | |
| 6 8 | = ⅓ | 115 | 13 | 4 |
| 1 8 | = ¼ | 28 | 18 | 4 |
| £3 17 4 | | £1359 | 1 | 8 |

CASE 8. Sometimes the sum is such, that by finding for one higher and subtracting the part too much, the process is shorter.

Ex. 347 at £3 4s. 11d.

| | | | | |
|---|---|---|---|---|
| | 347 | | | |
| | 3 | | | |
| | 1041 | | | |
| 5s. = ¼ | 86 | 15 | | |
| | £1127 | 15 | | But this is 1d. each too much, therefore we subtract 347 pennies. |
| 347d. or | 1 | 8 | 11 | |
| | £1126 | 6 | 1 | |

CASE 9. When the price is an even number of shillings, multiply by half the number of shillings, mark off all but

the last figure in the quotient as pounds, and double the last for shillings.

Ex. 347 at 8*s*.

| | | |
|---|---|---|
| 347 | | |
| 4 | | |
| —— | | £ *s*. |
| 138,8 | = 16, or | 138 16 |
| —— | | |

The reason of this rule is, we get only half the shillings in the true price, or a number of two shillings, which should be doubled to get shillings. Hence there are only ten in the pound; marking off the last figure is dividing by ten, and doubling it is making the remainder shillings.

When the price is an odd number of shillings, we might take the next number below, which will be even, and add once the top line after striking off.

Ex. 347 at 17*s*. per yard.

| | | £ *s*. |
|---|---|---|
| 347 | | 277 12 |
| 8 | 347*s*. = | 17 7 |
| —— | | ——— |
| 277,6 = 12 | | £294 19 |

Since 16 + 1 = 17, and 16 is 2 × 8.—The price at 16*s*. is too little by the price at 1*s*., we therefore add £17 7*s*.

## Case 10. When the number is not exact.

Ex. $347\frac{5}{16}$ at £3 5*s*.

| | | | |
|---|---|---|---|
| | 347 | | £ *s*. |
| | 3 | $\frac{4}{16} = \frac{1}{4}$ | 3 5 |
| | —— | | ——— |
| | 1041 | $\frac{1}{16} = \frac{1}{4}$ | 0 16 3 |
| 5 = $\frac{1}{4}$ | 86 15 | | 4 $0\frac{3}{4}$ |
| $\frac{5}{16}$ = | 1 0 $3\frac{3}{4}$ | | ——— |
| | ——— | | £1 0 $3\frac{3}{4}$ |
| | £1128 15 $3\frac{3}{4}$ | | |

Since $\frac{5}{16} = \frac{4}{16} + \frac{1}{16} = \frac{1}{4} + \frac{1}{16}$ Writing down the price of one, and taking as many parts of it as required, and adding the result to the price of the exact number.

Ex. 347 yds. 2 ft. 3 in. at same price.

| | | | |
|---|---|---|---|
| | 347 | | £ *s*. |
| | 3 | | 3 5 |
| | —— | in. | ——— |
| | 1041 | 18 = $\frac{1}{2}$ | 1 12 6 |
| 5*s*. = $\frac{1}{4}$ | 86 15 | 9 = $\frac{1}{2}$ | 16 3 |
| 2ft. 3in. = | 2 8 9 | | ——— |
| | ——— | | £2 8 9 |
| | £1130 3 9 | | |

Writing the price at one yard, and taking parts of it till we get the price of the required number of feet and inches.

All other denominations of weights and measures must be treated in exactly the same way.

Ex. 347 cwt. 3 qr. 7 lb. at £3 5s. a cwt.

| | | | | |
|---|---|---|---|---|
| qr. lb. | | £3 | 5s. | |
| 2 | = ½ | 1 | 12 | 6 |
| 1 | = ½ | | 16 | 3 |
| 7 | = ¼ | | 4 | 0¾ |
| 3 7 | | 2 | 12 | 9¾ |

347 cwt. at £3 5s.,
by Case 10 = 1127 15

£1130 7 9¾

Ex. 347 barrels 28 gall., at £3 5s. a barrel.

| | | | | |
|---|---|---|---|---|
| gal. | | £3 | 5s. | |
| 18 | = ½ | 1 | 12 | 6 |
| 9 | = ½ | | 16 | 3 |
| 1 | = ⅑ | | 1 | 9½ |
| 28 | | 2 | 10 | 6½ |

347 at £3 5s. = 1127 15

£1130 5 6½

Ex. The rent of 4 acres 2½ roods, at £5 10s. an acre.

| | | | | |
|---|---|---|---|---|
| a. r. | | £5 | 10s. | |
| 4 | | | 4 | |
| | | 22 | 0 | |
| 2 | = ½ | 2 | 15 | |
| ½ | = ¼ | | 13 | 9 |
| 4 2½ | | £25 | 8 | 9 |

The interest for 5y. 3m. 2w., at £17 10s. a year.

| | | | | |
|---|---|---|---|---|
| y. m. w. | | £17 | 10s. | |
| 5 | | | 5 | |
| | | 87 | 10 | |
| 3 | = ¼ | 4 | 7 | 6 |
| 2 | = ½ | 2 | 3 | 9 |
| 5 3 2 | | £94 | 1 | 3 |

There are many artifices which may be used in particular instances, but those given are the best general rules, and are sufficient to train to the acuteness necessary for particular cases.

One artifice, however, may be mentioned as sometimes of use for the price, or that part of it which is in pence and farthings. Consider the next lower number of dozens in the required number to be bought, make the pence shillings, and multiply the farthings by 12, then calculate the price for the number of dozens at this rate, and add to it the price of the part over, if any, taking care to remember the rate is now the original price.

When greater exactness is required, it is better to write down the number of farthings nearest to the true price, or to express the remainder after pence as fractions of a penny, and add all up at last.

## TARE AND TRET.

By the preceding rules we can calculate deductions to be made from the gross weight of goods, on account of the cases being weighed with them, on things liable to waste, or for any other cause.

**Ex. 1. Deducting 2 lb. 3 oz. avoirdupois per case, what will be the deduction on 255 cases.**

| | | | |
|---|---|---|---|
| lb. | | 255 | |
| 2 | | 2 | |
| oz. | | 510 | |
| $2 = \frac{1}{8}$ | | 31 | 14 |
| $1 = \frac{1}{16}$ | | 15 | 15 |
| 2 3 | 28 | 7)557 | 13 |
| | | 4)79 | 4 |
| | | 4)19 | 3 |

25

4 cwt. 3 qr. 25 lbs. 13 oz.

The difference from the cases in Practice is merely that the denomination of the upper line is now some weight and not money, the parts taken are of weights, and the quotients will be in the denomination of the upper line.

The sum obtained must be subtracted from the gross weight.

This deduction is called Tare.

Find the deduction on 4 tons 12 cwt. 2 qrs. at 4 lbs. per 104 lbs.

| Ex. 2. | cwt. | qrs. | lbs. |
|---|---|---|---|
| $4 = \frac{1}{26}$ | 92 | 2 | |
| | Cwt. 3 | 2 | 6 |

Since there are 4lbs. in 104lbs., there are 4 of any denomination in 104 of the same denomination, or 1 in 26.

Find the deduction on 2 tons 15 cwt. 2 qrs. 20 lbs., allowing 12 lbs. per cwt.

| Ex. 3. | cwt. | qrs. | lbs. |
|---|---|---|---|
| | 55 | 2 | 20 |
| $8 = \frac{1}{14}$ | 3 | 3 | 25 |
| $4 = \frac{1}{2}$ | 1 | 3 | 26 |
| 12 | Cwt. 5 | 3 | 23 |

Since 8 in 112 is one in 14, and 4 is the half of 8.

This deduction is that called Tret. The name is now obsolete in practice.

Subtracting from the upper line, we have 49 cwt. 2 qrs. 25 lbs. When the deduction in example 1 has also to be made, this deduction must be calculated afterwards.

It is, in most cases, better to compute these deductions by the rule of Simple Proportion.

## QUESTIONS.

A 1. What does Practice contain? 2. What is the rule when the price is less than 1*d.*? 3. In what denomination will the quotient be? 4. In what the remainder? 5. What are the two methods when the price is less than 1*s.*? 6. What will the remainder be when the number is written down as shillings? 7. What are the two methods when the price is more than 1*s.* and less than £1? 8. When the price is more than £1, how is the amount found? 9. What will the remainders be when the number is written down as pounds? 10. Give the general rule when the first line is at the rate of one in the next higher denomination? 11. When it is written down as the price of one in the highest denomination mentioned? 12. What denomination will the quotients be? 13. What the remainders? 14. What are the figures standing opposite each part? 15. When the price is such that we cannot take successive parts of each rate, what are the methods? 16. How can we find for one rate better, sometimes, by calculating for one higher? 17. When the price is an even number of shillings, what other method is there? 18. What must we do with the last figure? 19. How can this rule be applied to an odd number? 20. When the number whose price we want is not an exact number, how must we work?

B 1. What do we find by the rule of Tare and Tret? 2. What is the difference between thismanner of working, and that where money is concerned? 3. How is it generally better to calculate these deductions?

# CHAPTER VII.

## RATIO AND PROPORTION.

64. By ratio we mean the relation between two quantities of the same kind with respect to magnitude; and we have to consider by how much the first of the two is greater or less than the other, that is, what multiple part or parts one is of the other, or what fraction the less is of the greater.

A ratio is written with two dots placed between the terms which compose it, as 3 : 4. The first term is called the antecedent, the second the consequent.

65. Ratios can only enter into arithmetic as composed of numbers: thus, the ratio of one line to another, considered arithmetically, is the ratio of the number of units of length (see Art. 16, note) in that line to the number in the second.

66. Yet the quantities which compose the ratio must be of the same kind; there can be no ratio between any such quantities as 3 inches and 4 shillings: we can compare 3 as an abstract number to 4, but we cannot say 3 inches are greater or less than 4 shillings; there is nothing in common between 1 shilling and 1 inch, but in the ratio of 3 inches to 4 inches, we observe, there is something in common: the first contains a line of 1 inch 3 times, the latter 4 times. Hence, comparing them with respect to magnitude, we see they are as the numbers 3 and 4; and the comparison between them would convey the same impression, if we contrasted them

through the conception of the size of any other unit of length, as their relative value would still be the same.

67. But this unit must be the same in each term ;— 3 inches cannot be compared to 4 feet, as to number, but 4 feet must be represented as 48 inches ; and then 3 inches may be compared to 4 feet, or 48 inches, as the numbers 3 and 48.

68. Hence we conclude that the ratio of any concrete quantity to another of precisely the same kind, is the same as that between the two abstract numbers which express the number of units in each.

69. The ratios 3 : 4, 6 : 8, 9 : 12, 12 : 16, &c., are all manifestly equal, and each one is obtained from any other of the series, by multiplying or dividing its terms by the same number. Therefore, the terms of a ratio follow the same laws as those of a fraction ; hence, we write a ratio as a fraction, with the antecedent as numerator, the consequent as denominator.

70. Two ratios are said to be equal when the fractions which represent them are equal ; that is, when the first antecedent is the same multiple part or parts of the first consequent, that the second antecedent is of the second consequent.

The equality of ratios is called proportion : the four terms are called proportionals ; and when not written as forming two fractions, are represented by 4 dots placed between the two ratios, as 3 : 4 :: 6 : 8. The first and last terms are called extremes, the two others, means.

71. The terms of the 2nd ratio may be altogether different from the terms of the 1st ; for, as we have just seen, every ratio is equivalent to another whose terms are

abstract numbers. Thus, the ratio of 3 inches to 4 inches being precisely the same as the ratio of 3 to 4, and the ratio of 6 shillings to 8 shillings as the ratio of 6 to 8, they are equal, since 3 : 4 :: 6 : 8.

72. As the terms of each ratio must be of the same kind, we can only have two kinds of quantities in the same proportion; but as there is now actual numerical equality, we can multiply or divide the terms as mere numbers, or transpose in any way we please, following the same laws as in fractions; thus, we can interchange each antecedent or numerator with the consequent or denominator of the other, and can multiply or divide the 1st and 2nd, or the 1st and 3rd (since the 2nd and 3rd are interchangeable) by the same number.

73. Hence, multiplying both sides by the product of the two denominators (that is, the consequents), the first antecedent × 2nd consequent = the 2nd antecedent × 2nd consequent, or the extremes = the means.

Hence, to find an extreme we divide the means by the other extreme; to find a mean, the extremes by the other mean; or, to find a 4th proportional 4th term = $\frac{\text{3rd term} \times \text{2nd term}}{\text{1st term.}}$ with like expressions for the other terms.

74. Similarly, if we are given 4 numbers, such that the product of two of them is equal to the product of the others, they will be 4 proportionals, and the terms of each product will be the antecedent of one, and consequent of the other ratio.

Now, we observe that if one of these numbers be increased in any ratio, that multiplied into it must always be

decreased in the same ratio, as their product must always be the same.

75. Hence, if we have 4 quantities of the same kind, or the 1st and 2nd be of one kind, the 3rd and 4th of another, and an increase or decrease of the 1st or 2nd cause an increase or decrease of the 3rd or 4th, always in the same ratio, the 4 quantities will form a proportion; of which the 1st and 2nd will be 1st antecedent, and 1st consequent, or 1st and 2nd terms, and the 3rd and 4th 2nd antecedent and 2nd consequent, or 3rd and 4th terms, and the 4 quantities are called *direct* proportionals. But if an increase or decrease of the 1st or 2nd cause a decrease or increase of the 3rd or 4th always in the same ratio, the 1st and 2nd will be 1st antecedent and 1st consequent, or 1st and 2nd terms, but the 3rd and 4th will be 2nd consequent and 2nd antecedent, or 4th and 3rd terms, and the quantities are called *inverse* proportionals, and will evidently be *direct* proportionals when written down in the order, 1st, 2nd, 4th, 3rd, or 2nd, 1st, 3rd, 4th.

So that when we have 4 quantities so related that an increase or decrease in the same ratio is always observed between them after writing down two of the same kind (or if we are to find a 4th, observing that when written down it will be greater or less than the 3rd), so that the less precede the greater, or the greater the less; the other two must be written down in the same order, and we shall then have arranged the terms of a proportion.

---

## QUESTIONS.

A 1. What is ratio? 2. How is it written? 3. How many terms has it? 4. What are they called? 5. How

do ratios enter into arithmetic? 6. What is the ratio of one line to another? 7. Must the quantities which compose a ratio be of the same kind? 8. State the reason, and give an instance? 9. Is the ratio between 3 inches and 4 inches the same as that of 3 to 4? 10. Should the ratio be altered if the unit were altered? 11. Must the unit be the same in both terms? 12. Can 3 inches be compared to 4 feet as to number? 13. How must 4 feet be expressed that this may be possible? 14. To what ratio between abstract numbers will any ratio be equivalent. 15. What ratios are equal to that of 3 : 4? 16. Why may a ratio be written as a fraction? 17. Which term will be the numerator?

B 1. When are two ratios said to be equal? 2. What is this equality called? 3. State how the ratios are then written. 4. Which terms are extremes, and which means? 5. Must the terms of the 2nd ratio be of the same kind as those of the 1st.? 6. Explain this. 7. How many kinds of quantities may there be in the same proportion? 8. How can the terms be multiplied, divided, or transposed? 9. Which terms may we not divide by the same number? 10. How may the 2nd and 3rd terms be arranged? 11. To what is the product of the extremes equal? 12. How must a 4th proportional be found? 13. When the product of two numbers is equal to the product of two, will they form a proportion? 14. How will they be written? 15. What change will there be in one when that multiplied into it is increased or decreased? 16. When will four quantities be directly proportional? 17. When inversely? 18. How must they be written to be directly proportional?

## SIMPLE PROPORTION.

76. It remains to see how these principles are applied to the cases occurring in arithmetic.

The most important and general case is that in which we are given the price of a certain number of things, and are required to find the cost of any other number of the same things *at the same rate*, or to find how many another given price will buy.

Now, by the nature of the case, the rate is the same, that is, the number of things must increase *in the same ratio* as the price; if the price be doubled, trebled, or increased any number of times, the number of things will be doubled, trebled, or increased the same number of times.

Hence, the 4 quantities are proportionals: the first number of things bought, and their price, being the 1st and 2nd antecedents; the second number of things, and their price, the 1st and 2nd consequents; and to find any one of the terms is the same as finding a 4th proportional.

The same remarks apply to questions in Interest. The interest for the given sum is to be at the same rate as the interest for one hundred. Hence, the 4 are proportionals. Also, the interest for any time must be to the interest for any other time, as the times are to each other, or the 4 are proportionals.

Discount, being precisely of the same nature, needs no explanation.

The same considerations apply directly to Profit and Loss, the profit or loss on any two amounts, and the amounts themselves,

must be proportionals, the amount and its profit being the two antecedents.

Similarly in Stocks, the amount which can be bought for any price will increase in proportion to the price.

In Simple Partnership each partner's gain must be to the entire gain as his share of the capital to the entire capital.

Compound Partnership may be thus considered: the partner who has a sum invested for any period of time, ought manifestly to receive double or treble the sum to be received by another who has the same sum invested for only half or third the period. Similarly when the sum is doubled as well as the time, it will be equal to a sum four times as large for half the time, and so on for any increase of sum or increase of time; so that if we conclude that a sum for any time will be equal to any sum in any proportion less, providing the time be increased in the same proportion.

Hence, we see if we multiply the sum of each partner by the time, and add all these sums together, the gain of each will be in the same proportion to the entire gain as the sum thus found to the entire sum.

Similarly in Equation of Payments, the use obtained (or the interest) of the sum paid at once must be as much as the use of of all the sums, and since the use of any sum is double or treble the use of the same sum for half, third, &c., the time, it follows that we should estimate rightly by multiplying each sum by the time it was used; and that dividing the entire sum thus obtained by the amount to be paid, we should obtain the time in which its use would be equal.

*77.* The cases we have so far considered, are those in which the quantities are directly proportional, that is, in which if the price or rate be increased, the result is increased.

We come next to a class of questions slightly different, where the quantities, though proportionals, are not in this direct sense. Thus, if we are asked "if 5 men mow a

field in 6 days, how long will it take 3 men to mow the same field?"

Now, manifestly, the number of men and their time of work vary together, but an increase in one causes a decrease in the other. But we observe, that as the work is equal, the labour employed must be the same however estimated. Hence, the product of the agents and the time they act (when we have expressed like things in the same denomination) must be the same. Therefore the agents and the time must be so proportionals that the agent must be the antecedent of one ratio, and the time the consequent of the other, since the extremes in a proportion equal the means.

Exactly the same in Barter, when goods are exchanged at a certain price each article, for others at a different price, the entire value of both must be the same; that is, the number to be received will be less in proportion to the worth, therefore the four quantities will form a proportion in which the number of things and the value of each will be the antecedent of one ratio, and consequent of the other; since, then, the number and rate will be multiplied together.

We have now considered all the different classes of questions, in which we are given three to find a fourth, and have found them in all cases proportionals.

Questions like these form what is sometimes called the Rule of Three Inverse.

78. Hence, writing down the term of the same kind as the answer in the third place (that is antecedent of the second ratio), the greater or less of the other two. 2nd (that is consequent of the first ratio), as the answer will

be from the nature of the case, greater or less than the third, and put the other quantity first (or antecedent of the first ratio), multiply the second and third together (that is the means), and divide by the first term (one of the extremes) to get the answer (the other extreme).

But before we can multiply or divide, we must reduce the 1st and 2nd to the same denomination (67).

Now, in any proportion, we may divide the 1st and 2nd terms, and also the 1st and 3rd, by the same number without altering the value (72). Hence, we may strike out factors in 1st and 3rd or 1st and 2nd terms.

In Interest, the interest of the two different sums will be terms of the same kind, that is, terms of the same ratio.

In Profit and Loss, the profit on each amount stated.

In Stocks, the amount of stock bought, and so on in all cases, the two other quantities forming the two other terms.

It will be observed, however, that in Barter the terms of the first ratio will sometimes be of different kind, as measures may be bartered for weight. But we are given in each case the price of *a unit* in a certain denomination. Hence the quantities in the first ratio are considered as abstract numbers.

79. The property of proportion, that we may interchange the 2nd and 3rd term, is of use in cases where the 4 terms represent things of the same kind, as money, when the 1st and 3rd terms are of lower denomination than the 2nd.

For preserving the usual order, we shall have to reduce the three terms, and the quotient would be in the reduced denomination, but in the other case the 3rd term need not be reduced, and the quotient would consequently be in a higher denomination.

80. There is also a class of questions in which we have

more quantities involved: as men performing a certain work in days of certain length; and there are other quantities of the same kind referring to another work, but one element is wanting.

Now, when we made our units of time the same as before, the number of agents multiplied into the time they act will correctly represent the labour employed. And the products thus produced must have the same ratio to each other as that which exists between the work and the result of the labour; therefore, from this proportion we can find either the entire labour to be employed in the next case, which must be divided by the agents, or time given to obtain the quantity we want; or, if the entire labour be known, its effect. Similar considerations apply when the question is not one of work performed.

These questions form Compound Proportion, or, Double Rule of Three.

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## QUESTIONS.

A 1. Explain how goods at the same rate and their price are proportional. 2. Which will be the 1st term? Which the 3rd term? 3. The 2nd and the 4th term? 4. Shew how principal and interest proportionals. 5. What quantities will form each ratio in this case? 6. Explain how the interest for any time is found. 7. How are questions in Profit and Loss solved by proportion? 8. Which terms will be 1st and 3rd? 9. How question in stocks? 10. Explain Simple Partnership. 11. Why do we multiply the amount by the time in compound partnership? 12. How do we then obtain

the respective gain? 13. Explain, in Equation of Payments, how the use of the same sum is proportional to the time. If the sum be doubled, and the time trebled, how much will the use be increased?

B 1. How can agents and time be connected in a proportion? 2. If the agent be the antecedent, what term will its time be? 3. How are questions in Barter reduced to proportion? 4. If the rate form one term, which term will be the number at that rate?

C 1. Give the general conclusion in all cases. 2. Why is the reduction in 1st and 2nd term necessary? 3. Why may factors be struck out, and in what terms? 4. Explain when may we interchange the 2nd and 3rd term, and the advantage derived.

---

81. BARTER.—How many pounds of coffee, at 2*s.* 3*d.* per lb., must I receive for 45 yards of cloth, at 5*s.* 7*d.* per yard.

```
 s. d.      s. d.        yards.
 2  3   :   4  7    ::    45
 12         12
 ——         ——
 27         55
            45
            ——
           275
          220
          ————
Since 3 × 9 = 27 we    3)2475
  may use Rule 4 in     ————
  Division.             9)825
                      ——————————
                   Lbs. 91  10  10⅔
```

Since 4*s.* 7*d.* is the price of 1 lb., the quotient will be in a number of units of that denomination.

INTEREST.—What will be the interest on £350 for 3⅕ years at 5 per cent.

```
 £      £        £     £  s.        yr.  yr.      £  s.    £
100  :  350  ::  5  :  17 10         1 : 3⅕  ::  17 10 : 56
         5                                            3⅕
       ————                                        ——————
 1,00) 17,50                                       52 10
       ————                                      ⅕  3 10
      £17 10                                       ——————
       ————                                       £56  0
```

It is seldom of use to strike out factors in the first term when it is 100.

COMMISSION.—Find the commission on £240 at $1\frac{1}{4}$ per cent.

| £ | | £ | | £ | | £ |
|---|---|---|---|---|---|---|
| 100 | : | 240 | :: | $1\frac{1}{4}$ | : | 3 |

$$\begin{array}{r} 240 \\ 1\frac{1}{4} \\ \hline 240 \\ \frac{1}{4}\quad 60 \\ \hline 1{,}00)\,3{,}00 \\ \hline £3 \end{array}$$

DISCOUNT.—Find the discount on £2050, due 6 months hence, money being worth 5 per cent.

By Art. 79.

| £ | | £ | | £ | | £ |
|---|---|---|---|---|---|---|
| $102\frac{1}{2}$ | : | 2050 | :: | $2\frac{1}{2}$ | : | 50 |
| 2 | | 5 | | 2 | | |
| 205 | | 205)10250 | | 5 | | |
| | | 50 | | | | |

Or by striking out 205 between 1st & 3rd.

| £ | | £ | | £ | | £ |
|---|---|---|---|---|---|---|
| 1 | : | 5 | :: | 10 | : | 5 |
| | | | | 5 | | |
| | | | | 50 | | |

Since £100 at interest would produce £102 10s. at 5 per cent. in six months, therefore £2 10s. is the discount on £100. Calculating the discount as interest, we should have obtained £51 5s.

STOCKS.—The price of £100 Stock, 3 per cent., bring £96, what will be the price of £10,000 Stock, and the rate per cent. obtained.

| Stock | | Stock | | £ |
|---|---|---|---|---|
| 100 | : | 10,000 | :: | 96 |
| 1 | : | 100 | :: | 96 |
| | | 96 | | |
| | | 9,600 | | |

Dividing 1st and 2nd by 100.

| £ | | Stock | | £ |
|---|---|---|---|---|
| 96 | : | 100 | :: | 3 |
| 8 | : | 25 | :: | 1 |
| | | 1 | | |
| | | 8)25 | | |
| | | £3 2 6 | | |

Dividing 1st and 2nd by 4, and 1st, 2nd, and 3rd by 3.

PROFIT AND LOSS.—

Find the profit per cent. on selling goods bought at 3s. 6d., at 4s. 3d.

3s. 6d. : £100 :: 9d.

By Art. 79. 3s. 6d. : 9d. :: £100

$$\begin{array}{r} 3s.\ 6d. \\ 12 \\ \hline 42 \end{array}$$

7 : 3 :: 50

$$\begin{array}{r} 50 \\ 3 \\ \hline 7)\,150 \\ \hline £21\ 8\ 6\frac{6}{7} \end{array}$$

Dividing 1st and 2nd by 3, and 1st and 3rd by 2.

How must I sell goods bought at 5s. 8d. to realise 25 per cent.?

£100 : £125 :: 5s. 8d.

4 : 5 :: 12

$$\begin{array}{r} 12 \\ \hline 68 \end{array}$$

1 : 5 :: 17

$$\begin{array}{r} 17 \\ 5 \\ \hline 12)\,85 \\ \hline 7s.\ 1d. \end{array}$$

Dividing 1st and 2nd by 4.

PARTNERSHIP.—Three partners are in business, the chief partner has £4000, the second £2500, the third £1000; they gain £2000; find their respective gains.

| | | | £ | s. | d. | |
|---|---|---|---|---|---|---|
| 4000 | $\frac{4000}{7500} \times 2000 =$ | | 1066 | 13 | 4 | 1st partner's gain. |
| 2500 | $\frac{2500}{7500} \times 2000 =$ | | 666 | 13 | 4 | 2nd partner's gain. |
| 1000 | $\frac{1000}{7500} \times 2000 =$ | | 266 | 13 | 4 | 3rd partner's gain. |
| 7500 | | | £2000 | 0 | 0 | |

COMPOUND PARTNERSHIP.—Four partners are in business, one with £200 for twelve months, the second with £600 for six months, the third with £500 for six months, and the fourth with a £1000 for three months; they gained £300; find the respective shares.

| | | |
|---|---|---|
| $200 \times 12 = 2400$ | $\frac{2400}{12000} \times 300 =$ | £60 the 1st partner's gain |
| $600 \times 6 = 3600$ | $\frac{3600}{12000} \times 300 =$ | £90 the 2nd „ |
| $500 \times 6 = 3000$ | $\frac{3000}{12000} \times 300 =$ | £75 the 3rd and 4th „ |
| $1000 \times 3 = 3000$ | | |
| 12000 | | |

EQUATION OF PAYMENTS.—£500 is due to be paid at the following times: £100 in one month, £200 in six months, £150 in twelve months, and the remainder in eighteen months; when may all be paid at the same time?

| | | | | | | |
|---|---|---|---|---|---|---|
| $100 \times 17 = 1700$ | since the | 1st | is paid | 17 | months | before the last. |
| $200 \times 12 = 2400$ | „ | 2nd | „ | 12 | „ | „ |
| $150 \times 6 = 200$ | „ | 3rd | „ | 6 | „ | „ |
| 5000 | | | | | | |

And $\frac{5000}{500} = 10$ months before the last payment is the equated time.

COMPOUND PROPORTION.—5 men mow 4 acres in 6 days: how long will it take 8 men to mow 16 acres? Here $5 \times 6 = 30$ represent the days' work, and we must find a number of days' work that 30 may be to it as 4 is to 16. ∴ the number equals $\frac{30 \times 16}{4} = 120$; but as there are 8 men, there $\frac{120}{8} = 15$ will give the days required.

If the days were of a different length, the only difference would be, that we should have to find the number of hours' work.

## PROPORTIONAL PARTS.

82. To divide a given quantity into two or more parts, such that they may be to each other in the ratio of certain numbers.

Here each part will be to the given quantity as the number corresponding to it, to the sum of the numbers.

Ex. Divide 121 acres into two parts, in the ratio of 4 to 7.

$$\begin{array}{lrl} \text{Here} & 4 + 7 = 11 & \\ \text{and} & 11 : 4 : & 121 \\ & 1 : 4 :: & 11 \\ & & 4 \\ \hline & & 44 \end{array}$$

Also 121 − 44 is the other, or 77.

## ALLIGATION.

83. By Alligation we find how to mix different articles at various prices to sell the mixture at a given price.

Case 1. To find, when two or more quantities of different prices are mixed together, at what rate the whole must be sold to realise the same amount.

Here, if we find the value of the entire composition, and also how much is mixed together; and divide by this number, we shall have the rate at which all is to be sold.

---

83. I must apologise for the novelty, as far as I am aware, of these rules; I have given them this form, because at each step of the process, the reason is apparent, and an error easily detected. The answers in such cases are various, and every mode of working would most probably give a different result. My rules will therefore seldom give answers which are found in other arithmetics.

Ex. If 48 pounds of tea, at 3*s*. 6*d*. per pound, are mixed with 24 pounds, at 4*s*., what must the mixture be sold at?

Here the value of the 48lbs. is 168*s*.
24lbs. is 96*s*.

72 264

Therefore, the price per pound is $\frac{264}{72} = \frac{11}{3} = 3s.\ 8d.$

CASE 2. To find how to take two quantities of different price to form a mixture at any mean rate.

RULE. Find the difference between the price of each and the required price, and take of each sort a number equal to the difference for the other.

The reason of the rule is, the loss on selling the higher priced too low, must be counterbalanced by the gain on the other; and, as the first difference is greater or less than the second, we must take less or more of the first quality than the second, or the difference, and the numbers will be inversely proportional.

Ex. How much at 4*d*. and 9*d*. must be taken to sell the mixture at 6*d*. per lb?

Gain of 1lb. of 1st at 4*d*., sold at 6*d*., is 2*d*.; of 3lb. is 6*d*. gain.
Loss on 1lb. of 2nd at 9*d*., sold at 6d., is 3*d*.; of 2lb. is 6*d*. loss.
Therefore 3 of the first and 2 of the second give the proportion in which they are to be taken.

CASE 3. I. To find how to mix any number of articles, at different prices, to form a mixture at a given mean rate. Find the gain or loss on selling one of any kind; and how much of each of the others must be taken to produce the same gain or loss. Multiply each on which there is gain by the number on which there is loss, and each on which there is loss by the number on on which there is gain: the products will be the respective quantities to be taken.

---

I have placed the reasons immediately after the rules, as there can be no advantage derived unless the process be understood.

Case 2 is included in this rule.

II. If the mixture must also be equal to a certain number, add all these products together, divide the number by it: the products multiplied by the quotient will give the required amount.

The reason of Rule 1. is, that we make the gain or loss on each exactly the same, therefore the total gain so found will be to the total loss as the number on which there is gain to the number on which there is loss, or they will form a direct proportion; therefore, the gain multiplied by the number on which there is loss, will give the same result, that is, will counterbalance the loss multiplied by the number on which there is gain.

The 2nd rule is derived, because when we have once ascertained the number of each kind to be taken, we may increase or diminish all in the same proportion.

Ex. Required a mixture at 8*s*. per lb. of the following. The 1st at 8*s*., the 2nd at 4*s*., the 3rd at 3*s*., the 4th at 9*s*., the 5th at 10*s*. Ex. 2. Also to make a mixture of 128lbs.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Here the gain on 1lb. of the | *s.* | | *s.* | Ans. | | *s.* | Ans. | | *s.* | |
| 1st sold at 8*s*. instead of | 6 | is | 2 | 2 | is | 4 | 20 | is | 40 | |
| $\frac{1}{2}$lb. of the 2nd ditto | 4 | „ | 2 | 1 | „ | 4 | 10 | „ | 40 | |
| $\frac{2}{5}$lb. of the 3rd ditto | 3 | „ | 2 | $\frac{4}{5}$ | „ | 4 | 8 | „ | 40 | |
| | | | 6 gain | | | 12 gain | | | 120 gain | |
| Loss on 2lb. of the 4th ditto | 9 | is | 2 | 6 | is | 6 | 60 | is | 60 | |
| 1lb. of the 5th ditto | 10 | „ | 2 | 3 | „ | 6 | 30 | „ | 60 | |
| | | | 4 | $12\frac{4}{5}$ | | 12 loss | 128 | | 120 loss | |
| | | | since 128 ÷ $12\frac{4}{5}$ is 10. | | | | | | | |

Now, we observe that we may take the first three in any proportion we please, and also the last three, providing we make the gain exactly equal to the loss; and when the entire quantity to be taken is fixed, multiply each so as to make up the same amount, that is, by the quotient of the number divided by their sum.

Since the number giving a different result to the given quantity is 3, and the same is 2, we multiply each of the former by 2, and divide by 3.

The 2nd rule is derived, because, when we have once ascertained the number of each kind to be taken, we may increase or diminish all in the same proportion.

CASE 4. When the mixture required must contain a certain quantity of another, at a given price.

I. Find the gain or loss on selling the given quantity; find the portions of the others which will give an equal gain or loss in each case. Multiply each gain by the number on which there is loss, and each loss by the number on which there is gain (the given quantity being included), and divide the result by the number by which we have multiplied the given quantity; the quotients will give the required portions.

N.B.—We need only multiply and divide the numbers giving a result different to the given quantity.

Ex. Find how many lbs. at $4d.$ must be mixed with others at $10d.$, $13d.$, and $18d.$ to sell with 20 at $6d.$, the entire mixture to be worth $8d.$ per lb.

| | lb. | | *d.* | | *d.* | | *d.* | Ans. | *d.* |
|---|---|---|---|---|---|---|---|---|---|
| Here the gain on | 20 | sold at | 8 | instead of | 6 | is | 40 | 20 | 40 |
| | 10 | „ | 8 | „ | 4 | „ | 40 | 10 | 40 |
| | | | | | | | 80 | | 80 gain |
| Loss on | 20 | „ | 8 | „ | 10 | „ | 40 | $13\frac{1}{3}$ | $26\frac{2}{3}$ |
| | 8 | „ | 8 | „ | 13 | „ | 40 | $5\frac{1}{3}$ | $26\frac{2}{3}$ |
| | 4 | „ | 8 | „ | 18 | „ | 40 | $2\frac{2}{3}$ | $26\frac{2}{3}$ |
| | | | | | | | 120 | 52 | 80 loss |

We observe that we may alter the second quantity, if we alter the other three, so as to make the loss equal to the total gain, and may take any numbers of the last three, if we take care the entire loss remain the same.

II. If the entire quantity be fixed, we must subtract the number of the given quantity, and one of the numbers which gives an opposite result, from the entire quantity. Make the total loss

and gain on the rest equal; divide the difference just found by their sum, and multiply each finally by the quotient.

Thus, in the Ex., if it were required to make a mixture containing 50 lb. Subtracting the given quantity and 8 lb., we have left 22 lb., and the gain and loss on the numbers excluded is the same.

Hence we have—

| lb. *d.* | lb. *d.* | Ans. *d.* *d.* |
|---|---|---|
| | | 20 at 6 is 40 gain. |
| | | 8 at 13 ,, 40 loss. |
| 10 at 4 is 40 gain. | 20 is 80 gain. | 10 at 4 ,, 40 gain. |
| 20 at 10 ,, 40 loss. | 20 ,, 40 loss. | 10 at 10 ,, 20 loss. |
| 4 at 18 ,, 40 loss. | 4 ,, 40 loss. | 2 at 18 ,, 20 loss. |
| | 44 80 loss. | 50 40 loss. |

It is evident the numbers 8, 10, 10, 2, can be varied in any proportion, subject to the two conditions that there be no final loss, and their sum, together with 20, be 50.

## POSITION.

84. Case 1. To find a number which, added to certain parts of it, will make up a given number.

Here, if we take one as often as we are to take the required number, add the same parts required of it, and divide the given number by the result, the quotient will be the required number.

Ex. find a number which, added to its half and its third, will give 121.

Here $1 + \frac{1}{2} + \frac{1}{3} = \frac{11}{6}$ and since the required number multiplied by this will give 121, therefore the required number is $121 \div \frac{11}{6} = 121 \times \frac{6}{11} = 66$.

Case 2. When two or more are to share a certain sum, and their shares are not exact parts, take unity as the share of one of them; take parts of this, and

write down the sum by which each of the rest exceeds or is less than each of these parts; add one and all its parts together, and the sums by which each is different from the exact parts; subtract the latter from the quantity; and divide by the number representing the parts of 1. This will give the share supposed.

**Ex.** There are £250 to be divided between A., B., and C. B. receives as much as A., and £8 more; and C. one quarter as much as both together: what is each man's share?

Here if A. receives 1.
B. receives 1 and £8.
C. receives $\frac{1}{4}$ (2 and £8) or $\frac{1}{2}$ and £2.
And $1 + 1 + \frac{1}{2} = \frac{5}{2}$; £8 + £2 = 10.
$250 - 10 = 240$.

$$\text{A. receives } 240 \div \tfrac{5}{2} = \frac{240 \times 2}{5} = 48 \times 2 = £96.$$

Since $\frac{5}{2}$ times what A. receives makes up £240. Hence, B. receives £104, and C. receives £50.

The same method may be applied to other questions, but the working is generally more difficult.

Ex. A. is now twice as old as B.; eight years ago he was three times as old, and one year more: find the age of each.

Here if 1 be taken for A.'s age, $\frac{1}{2}$ will be B.'s age. $1-8$ years, will be A.'s age; $\frac{1}{2}-8$ years, B.'s age 8 years ago. Now, 3 times B.'s age + 1 year is $\frac{3}{2}$ − 24 years + 1 year, or $\frac{3}{2}$ − 23 years. But this is same as 1 − 8 years + $\frac{1}{2}$ − 15 years, and is equal to A.'s age, or 1 − 8 years. Therefore $\frac{1}{2}$ − 15 years must be equal to 0; or, $\frac{1}{2}$ = 15 years; hence A. is 30 years old, B. 15 years.

---

Care must be taken to keep 1 and its parts separate from other numbers; as in reality a number very different to unity is generally represented by it. It will be seen that the first case is merely a simple form of the second, and might be entirely omitted.

## EXCHANGE.

85. The coin of different kingdoms not being the same, though of the same material, the object of Exchange is to find how much ought to be received in a certain coin or currency, for any amount in another.

The rate of Exchange being given, we can calculate that for any other sum; for the sum to be given, and that to be received, must increase in the same proportion.

RULE. Reduce the coin of the same kind to the same denomination, and state as in Simple Proportion.

Ex. A franc contains 100 centimes, and £1 being worth 25 francs 35 centimes, how much must be received for £45 3*s*.

```
 £       £  s.       fr. ct.
 1   :  45  3  ::   25  35
20      20          100
--      ---        ----
20     903          2535
                     903
                   -----
                    7605
                  228150
                 -------
          2,0) 228910,5
                 -------
         1,00) 1144,55½
                 -------
               1144 55 neglecting the fraction;
```

therefore 1144 francs 55 centimes is the amount.

To change weights and measures we must proceed exactly by the same method.

Ex. Find how many miles there are in 328 kilometres, 4 kilometres making 2½ miles.

```
                  kil.    kil.       miles.
                   4  :   328   ::    2½
Dividing 1st and   1       82          2
 2nd terms by 4.            5          —
                        -----          5
                      2) 410
                        -----
                         205 miles.
```

## ARBITRATION OF EXCHANGES.

86. Sometimes it is more advantageous to exchange into another currency, through the medium of other countries or currencies, instead of directly. Arbitration of Exchanges is the mode of calculating the final result.

We may calculate for each currency in succession by the rule already given, or by the following: —

Place in the first series the amount to be given in the coin of the 1st country, for a given sum in the currency of the country next following, place the latter sum in the second series, and so continue, placing the second amount in any currency in the first series, and the first amount in the second series. Reduce like coins to the same denomination; find the product of all the terms in the second series, and divide by the product of the first, which has one term wanting; the quotient will be the answer, and must be compared with the amount which would have been received by direct exchange to ascertain the gain or loss.

Ex. A French franc being worth 9½*d.*, and 3 francs being worth 1½ stivers, find the gain or loss on remitting through France £38; a stiver being worth 1*s.* 8*d.*

| | | |
|---|---|---|
| | *d.* | *fr.* |
| | 9½ | 1 |
| | *fr.* | *st.* |
| | 3 | 1½ |
| Required stivers for | £38 | 38 × 240*d.* |

But by the rule the number is $\frac{38 \times 240 \times 1 \times 1\frac{1}{2}}{9\frac{1}{2} \times 3} = 480$

Now, by direct exchange, as each stiver is worth 20 pence, we shall have $\frac{38 \times 240}{20} = 456$ stivers, therefore the gain through France would be 24 stivers, or 24 × 20 pence, that is £2.

It will be seen that by this rule the amount in any currency is always divided by another amount in the same currency. Now, by the rule of direct exchange, the rate of exchange is always proportional to the amount exchanged; therefore the numerator of this fraction is equivalent to any amount in the same currency multiplied the sum which would be received for the first sum in another currency, divided by that for the second sum. Hence, we could express the first fraction by another, having the product of two or more amounts in different currencies, divided by the equivalent for each in another of the currencies forming the numerator. Hence the correctness of the rule.

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The same rule may be applied to ascertain the equivalent for any weight or measure, when not known directly, but through the medium of other countries.

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## QUESTIONS.

A 1. What is found by proportional parts? 2. What is the rule?

B 1. What is Alligation? 2. How must we find the price when there are two or more to be mixed? 3. How when two, but the price is fixed? 4. How must the rule be changed when there are several? 5. How when the entire quantity is fixed? 6. When there is a certain quantity whose price is fixed, what is the rule? 7. When the entire quantity is fixed? 8. Explain the variations we may make in each case.

C 1. What is found by Position? 2. Give the rule in the 2nd case.

D 1. What is Exchange? 2. What is the rule? 3. What is Arbitration of Exchanges? 4. Give the rule.

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# CHAPTER VIII.

## SQUARE ROOT.

87. The square of a number is the number multiplied by itself: thus, the square of 3 is 3 × 3, or 9.

When we are required to find a number which, squared, will give one proposed, the process is called Extraction of the Square Root.

It is found that if marks are placed to the left of every second figure, counting from the units place, so as to divide into periods of 2, and the last figure, if the

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87. To explain the rule for pointing: since the square root of 100 is 10; of 10,000 is 100; and of 1,000,000 is 1,000; &c., always adding two figures to the square to get another figure in the square root, and observing that these are the smallest numbers that can be written with the same number of digits, we perceive that the square root of any number between 1 and 100 has one figure, between 100 and 10,000 has two figures, between 10,000 and 1,000,000 has three figures, &c. That is, one or two figures in the square gives one in the root, and every two figures more give one in the root. Hence, if we point every second place the periods so formed will shew the number of figues in the square root.

We find any square number (as 625) can be put under a form (as 400 + 2 × 20 × 5 + 25) where the first term is the square of the highest number of tens contained in it, and the last term is the square of the remaining part of the root (400 is the square of 20, and 25 of 5, and 20 + 5 make up the entire root), and the second twice the product of the two parts; and we observe that we can arrive at the proper root by the following contrivance. If we take, as nearly as we can, the square root (2) of the first term (6), we see this will give us the number of tens we want. Now if we subtract the square of these (400) we shall have a remainder

number of figures be odd, be counted as one period, they will give the number of figures in the root.

Rule. Find the number which, squared, will give as nearly as possible the first period; place it in the quotient, and its square beneath the first period; subtract, and bring down the next period. Use as a divisor twice the figure in the quotient: divide the first figures by it; place the result (or if it be large, the number next lower) at the right of the quotient, and also of the divisor. Multiply the entire divisor by this number, and subtract the product. Bring down another period, double the figures in the quotient, and proceed as before.

| Ex. 1. | | Ex. 2. | |
|---|---|---|---|
| | 6'25(20 + 5 | | 7'84(28 |
| | 4 | | 4 |
| 40 + 5) | 225 | 48) | 384 |
| | 225 | | 384 |

---

(225 or 2 × 20 × 5 + 25), and, we remark, that we can obtain the other part of the root (5) by dividing the first figures (22) by twice the number of tens (4), and that if we use the new root (5) as a new figure in the quotient of an operation in division, and twice the root first found + the new root (2 × 20 + 5) as a divisor, we shall obtain a product which, subtracted from the remainder (225), will destroy it.

If there are more figures the only modification is, we must make the second quotient as great as possible, and continue as indicated.

There is one caution, however: sometimes the number found by dividing by the first figure is too large, as we not only have to divide the remainder into two parts, such that twice the old root multiplied by the new shall equal the first, but also the other part must be such that the old root squared will equal, or not be greater, than it. Both these conditions are fulfilled in the first Ex. by dividing 22 by 4, but in the second we must take one less than we should get by dividing 38 by 4, in order to fulfil them.

When we have doubled the number already found for a new divisor, and the remainder with the next period be not more than ten times greater, we must bring down another period, and write a cipher in the quotient, as in Ex. 4.

| Ex. 3. | Ex. 4. |
|---|---|
| 3'06'56(284 | '11'60'76'49(3407 |
| 4 | 9 |
| 48)406 | 64) 260 |
| 384 | 256 |
| 564) 2256 | 6807) 47649 |
| 2256 | 47649 |

When the square root of a decimal, or a whole number and a decimal, is required, we must put points after every second figure, counting from the decimal point, and add ciphers if necessary.

Or if the square root of a whole number cannot be found exactly, periods of two ciphers must be added.

Each period, in both cases, will give one decimal place in the square root.

Find the square root of 10·57347 to 4 decimal places.

```
'10.57'34'70'00'(3·2516
  9
62)157
   124
645)3334
    3225
6501)10970
      6501
65026)446900
       390156
        56744
```

It is evident when ciphers have to be brought down the operation will never terminate, as no number less than 10, multiplied by itself, will give a number ending in zero.

88. **To extract the square root of a fraction.**

**Rule 1. If both numerator and denominator are perfect squares, extract the square root of each, and divide the square root of the former by that of the latter.**

Ex. Extract square root of $\frac{9}{4}$.

Square root of 9 = 3, and of 4 = 2;

therefore, square root of $\frac{9}{4}$ is $\frac{3}{2}$.

**Rule 2. When they are not both perfect squares. Multiply the numerator and denominator by a number which will make the denominator a perfect square, and take the square root of the new numerator and denominator.**

**N.B. This number need not exceed the denominator, as that multiplied by itself will give a perfect square.**

Ex. Extract the square root of $\frac{3}{8}$.

Here, $2 \times 8 = 16$, a perfect square, and $\frac{3}{8} = \frac{6}{16}$ or $\frac{6}{4 \times 4}$ and the square root of $\frac{6}{16}$ is $\frac{2.449}{4}$ ... or .612 ...

```
        6  ( 2.449
        4
       ---
   44) 200
       176
       ----
  484)  2400
        1936
        -----
 4889)  46400
        44001
        -----
         2399
```

Rule 3. It is sometimes better to divide the numerator by the denominator, and take the square root of the quotient.

---

88. The reason of Rule 1 may be seen by considering that in multiplying fractions we multiply the numerators together, and also the denominators. Now, squaring the fraction we get by Rule 1, we should multiply the numerator by itself (that is, square it), and the denominator by itself. Hence the numerator must be the square root of the other, &c.

Rule 2 is merely an artifice : if we had the square root of

## CUBE ROOT.

89. The cube of a number is its square multiplied by the first number. Thus, the cube of 3 is $9 \times 3$ or $3 \times 3 \times 3$.

To extract the cube root of a number, we must find another which, cubed, will give the one proposed.

RULE. Place a point over every third figure, counting from the units place. Find, as nearly as possible, the cube root of the first period, place it in the quotient, followed by as many ciphers, less one, as there are periods;

---

the denominator a decimal of many places, it would be inconvenient to divide the square root of the numerator by, so we make the denominator a perfect square, and multiply the numerator by the same number, so as not to alter the value of the fraction.

89. Rule for pointing. The cube root of 1,000 is 10, of 1,000,000 is 100, of 1,000,000,000, is 1,000, &c. Hence every number of not more than three figures will have a cube root of one figure, every number of four, five, or six figures will have a cube root of two figures, every number of seven, eight, or nine figures will have a cube root of three figures. Therefore if we mark off counting from the units place, and mark off the 3rd, 6th, 9th, &c., (considering the figures between, a period,) that is, every third figure, this will give the number of figures in the root.

Pursuing the same method as in the square root, we may put 15625 under the form 8000, which is the cube of 20, and $6000 + 1500 + 125$ or $20 \times 20 \times 20 + 3 \times 20 \times 20 \times 5 + 3 \times 20 \times 5 \times 5 + 5 \times 5 \times 5$, and reasoning in the same way as before, we deduce that we must extract the cube of the first period; this will give the highest power of tens; divide the remainder by three times its square, place the result in the quotient, and use for a divisor three times the square of the root already found, + three times the product of the old root and new root, + the square of the new root, and for a quotient the new root.

subtract its cube; bring down the remainder; take three times the square of the quotient; consider how many times it is contained in the remainder, supposing ciphers to follow the 1st period; place the result (or a less number, if it be large) in the quotient; write as divisor three times the square of the root found before, + three times the product of the two parts of the root, + the square of the last part of the root: subtract, and proceed as before, if there be more than two periods.

Ex. 1.

```
                               15'625 (20 + 5, or 25
                                8000
                               ------
3 × 20 × 20 = 1200              7625
3 × 20 ×  5 =  300
      5 ×  5 =   25
             ------
              1525 )            7625
             ------            ------
```

Ex. 2.

```
                                   28'094'464 (300 + 00 + 4,
                                   27000000      or 304
                                   --------
3 × 300 × 300 = 270000              1094464
3 × 300 ×   4 =   3600
        4 ×   4 =     16
                --------
                273616 )            1094464
                --------           --------
```

Where there are decimals or ciphers brought down, we must point every third figure from the decimal point, and proceed as in whole numbers. There will be as many decimal places in the result as there are periods below the decimal point, and, as in square root, there will always be a remainder when ciphers are brought down.

To extract the cube root of a fraction, multiply the denominator (if necessary) so as to make it a perfect cube; or divide the numerator by the denominator, and find the cube root of the quotient.

QUESTIONS.

A 1. What is the square of a number? 2. What the square root? 3. How do we know the number of figures in the root? 4. What is the rule for its extraction? 5. How is the first figure found? 6. How the divisor and dividend for the second? 7. What is the complete divisor? 8. How will the third figure be found? 9. When will there be a cipher in the root? 10. How are the decimal places pointed? 11. Why will the operation never terminate when ciphers are attached? 12. How is the square root of a fraction found when the denominator is a perfect square? 13. When it is not?

B 1. What is the cube of a number? 2. What the cube root? 3. How do we point? 4. How find the first figure of the root? 5. How the second?

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## INVOLUTION AND EVOLUTION.

90. By Involution we find how much a number will amount to, multiplied by itself as often as required. The result is called the number raised to that power.

Thus, $3 \times 3 \times 3 \times 3 \times 3$, or 243, is called the 5th power of 3. The number representing the power to which it is to be raised is generally placed over the other, to the right hand (thus, the 5th power of 3 is written $3^5$), and called an index.

The process by which we arrive at the number, or as it is called, root, from the power, is called Evolution, or Extracting the Root, and that representing the power to which it has been raised is placed before the word root, or put as denominator of a fraction, and written as an index; thus, 3 is the 7th root of $3^7$, $3^{\frac{5}{7}}$ the 7th root of $3^5$. A symbol $\sqrt{}$ is often used instead of root, thus $\sqrt[5]{243}$ is the 5th root of 243.

It is laborious to find for the power when it is high, or when the number is large; and a general (though possible) rule for the extraction of any required root is too complicated for working. To find such roots as the 4th or 8th, we may extract the square root twice or thrice in succession; for the 6th, the cube root of the square root; for the 9th, the cube root twice. But the necessity of all operations of this kind is obviated by the use of Logarithmic Tables.

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## LOGARITHMS.

91. A Logarithm of a number is that power to which 10 must be raised to give the number.

It may seem difficult to conceive how 10 can thus produce all numbers, as it is evident that to obtain any one of the numbers below 10, it must really be divided, and not multiplied; but the word power here denotes that 10 can be so squared, cubed, &c., and such a root then extracted that we can arrive at any required number.

The index would consequently (except for 10, 100, 1000, &c.) be a fraction proper or improper, and may, therefore, always be expressed by a decimal.

These indices or the logarithms of all numbers between zero and 100,000, are registered in tables called Logarithmic Tables, and directions are given so that by a little calculation, or, as it is called, taking proportional parts, we may extend their use to any number whatsoever.

It is, however, only the decimal part of the logarithm which is registered (that is, the proper fraction); a whole number must be prefixed, to make the logarithm complete,

one less than the number of figures, not counting decimal places, of the number whose logarithm we want. If the number be between zero and 10, the part given is, however, the entire logarithm.

The tables are extended to mixed numbers and decimals by the following considerations :—all numbers whose digits are the same, not regarding the decimal points, have the same register; the whole number to be prefixed depends, however, on its position, and is determined as before explained; but if we are to find the logarithm of an entire decimal, the prefix will be negative, and equal to the number of ciphers between the first significant figure (that is, one of the numbers, 1, 2 ... or 9) and the decimal point.

This is called the characteristic figure, and when prefixed we obtain the entire logarithm.

92. These tables are chiefly of use in the following cases :—

1. It is found that to multiply two numbers together, if we add their complete logarithms, and find the number standing opposite the decimal part of the result, it will be the required product; marking off the decimal places, if required, by remembering it will have one figure more in the integral part than there are units in the integral part of the sum.

2. To divide one number by another, we must subtract the logarithm of the latter from that of the former, and use the tables as before.

3. To find any power of a number, we must multiply its logarithm by the number expressing the power.

4. To find any root, we must divide by that representing

the root to be taken, or multiply and divide when the index is a fraction.

We must take care in all cases to add, subtract, multiply, or divide the characteristic figures.

Hence, (as multiplication and division are addition and subtraction when the numbers are the same) all operations are reduced to addition and subtraction.

I have not attempted to prove any of the properties here asserted, the student having time fully to investigate the matter, would be better able to perceive the reasons when further advanced, and it would be to impede his progress to study them at this point.

The tables published by Taylor and Walton, extending as far as 10,000, will be found useful and sufficient for ordinary purposes. Under Art. 96 I have given an instance of the application of Logarithms.

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## QUESTIONS.

A 1. What is Involution? 2. What is meant by a number raised to any power? 3. How is this generally expressed? 4. What is meant by index? 5. What is Evolution? 6. How is the root often written? 7. What is the meaning of a fractional index? 8. How are these powers and roots best calculated.

B 1. What is a logarithm? 2. How does it appear that 10 can produce all numbers? 3. What are Logarithmic Tables? 4. How far do the tables extend? 5. What is the part registered? 6. What must be prefixed to it when the proposed number is a whole number? 7. A mixed number? An entire decimal? 9. Of what use are the tables in Multiplication? 10. Division? 11. Involution? 12. Evolution? 13. What care must be taken as to the characteristic figures? 14. What is the general conclusion?

# CHAPTER IX.

## ARITHMETICAL PROGRESSION.

93. When we have a series of numbers increasing or decreasing by a common difference, the series is called an Arithmetical Progression.

The difference is found by subtracting any term from the one immediately before it.

Rule 1. To find any term. Multiply the common difference by the number of terms, diminished by one, and add the product to the first term.

Ex. The 10th term of $2 + 5 + 8 + \ldots$ Here $5 - 2 = 3$ the com-difference, and $3 \times 9 + 2 = 27 + 2 = 29$, the 10th term.

Rule 2. To find the sum of series. Add the first and last terms together; multiply by the number of terms, and divide by two.

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(93.) Rule 2. Ex. The first term and last (2 and 53) make up 55, the second (5) the last but one (50) also make up 55, the third (18) and the last but two (47) also give 55, and so on for all the pairs of terms; but there are 18 terms, and therefore 9 pairs, or the sum is $9 \times 55, = 495$.

Again, supposing we wanted 19 terms of $2 + 5 + 8 +$ Here there would be 9 pairs, and the middle term, the 10th term, would be $2 + 9 \times 3 = 29 = \frac{58}{2}$, and 1st (2) and last (56) are 58, and so with the other 8 pairs, therefore we have $\frac{58}{2} + 58 \times 9 = \frac{58}{2} \times 19$.

The first term is less than the second term by the common difference. The last term is greater than the last but one by the common difference. Consequently, the sum of the first and last is equal to the sum of the second and last but one. Similarly this sum is equal to the sum of the 3rd and last but two, and so on;

Ex. 2 + 5 + 8 + ... to 18 terms. Here 53, by Rule 1, is the last term, and 2 + 53 = 55 is the sum of the first and last, therefore the sum is $\frac{55 \times 18}{2} = 495$

**Rule 3. To insert any number of terms between two numbers. Subtract one from the other, divide by the number of terms required, increased by one, the quotient will be the difference required.**

Ex. Insert 5 terms between 1 and 13.

Here 5 + 1 is 6, and 13 − 1 is 12, therefore the common difference is 12 ÷ 6, or 2, and the terms are 3, 5, 7, 9, 11.

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## GEOMETRICAL PROGRESSION.

**94. When we have a series of numbers, of which each contains the one next before a certain number of times,**

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hence if there be an even number of terms, there are half that number of pairs, each equal to the first and last, or half the product of the first and last by the number of terms.

Now, if there be an odd number of terms, there will be half that number, diminished by one, of pairs, and also a term midway between the first and the last, and therefore half their sum. These added together will give the same result as in the last case, namely, the first and last term multiplied by the number of terms and divided by 2.

(94.) Three times every term of 2 + 6 + 18 + 54 + 162 + 486 = 6 + 18 + 54 + 162 + 486 + 1458, or equals 2 + 6 + 18 + 54 + 162 + 486 + 1458 − 2, that is, the first series + 1458 − 2. Subtracting once, that is once every term of, the first series, we get twice every term of the series 2 + 6 + 18 + 54 + 162 + 486 = 1458 − 2.

Therefore $2 + 6 + 18 + 54 + 162 + 486 = \frac{1458 - 2}{2} = 728.$

Or generally, thus, multiplying the whole series by the common ratio would make every term one higher, therefore the new series

that is, increases in a common ratio, the series is called a Geometrical Progression.

1. To find the Common Ratio. Divide any term by that before it; this will give the common ratio.

2. To find any term. Multiply the common ratio by itself as many times, less one, as there are terms (that is, raise the common ratio to this power), and multiply the product by the first term.

3. To find the sum of a series. Find the term succeeding the last, take the difference between this and the first, and divide it by the difference between the common ratio and one.

Ex. Find the common ratio, the sixth term, and sum of the series; $2 + 6 + 18 + \ldots.$

Here $\frac{6}{2} = 3$, the common ratio; and $2 \times 3 \times 3 \times 3 \times 3 \times 3$, or $2 \times 3^5 = 486$, the sixth term; therefore, 1458, is the 7th term, and the sum is $\frac{1458 - 2}{3 - 1} = \frac{1456}{2} = 728.$

95. Compound Interest may now be conveniently calculated. The Amount of £1, at 5 per cent., at the end

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would begin with the 2nd term, and end with the term after the last. Hence, subtracting the original series, every term would go out, but the last of the higher series, which would be diminished by the first term of the first series.

Now we obtained this result by multiplying the first series by the common ratio (that is, adding the first series to itself that number of times), and subtracting from the product the first series. Hence there is left the first series, added to itself once less (that is, multiplied by a number one less) than the common ratio; but this must be precisely the same as the result derived by the first method. Hence the series equals the term succeeding its last term, diminished by its first term, and divided by one less than the common ratio.

of the year, is £1 + $\frac{1}{20}$ of £1 or $\frac{21}{20}$ of £1. At the end of the second year this will amount to $\frac{21}{20}$ + $\frac{1}{20}$ of $\frac{21}{21}$ of £1, or $\frac{21}{20} \times \frac{21}{20}$ of £1. At the end of three years we shall have $\frac{21}{20} \times \frac{21}{20} \times \frac{21}{20}$ of £1; and similarly for any number of years, multiplying $\frac{21}{20}$ by itself as often as there are years.

If now we considered £5, we should find this would amount to £5 + $\frac{1}{20}$ of £5, or $5 \times \frac{21}{20}$ in one year; $5 \times \frac{21}{20} \times \frac{21}{20}$ of £1 in two years, and so on; and for any number of pounds, for any time, we should find the number of pounds $\times \frac{21}{20}$ as often as there were years; or if payable oftener (as half-yearly), multiplied by a number representing £1, and its interest for that period (as $1 + \frac{1}{40}$ or $\frac{41}{40}$), as often as there were periods of payment. Similarly, if the interest were not 5 per cent., but any other, as 4, then, if payable yearly, the multiplier would be $1 + \frac{4}{100}$ or $\frac{26}{20}$, that is $\frac{13}{10}$; or half-yearly, then $1 + \frac{2}{100}$ or $\frac{51}{50}$; with similar changes for other periods and other rates of interest, but always one multiplier throughout, namely, 1 with its interest at the given rate for the first period; and the amount would be the principal multiplied by this as often as there were periods. Hence, finding the amount of any sum at compound interest, payable yearly, or any number of times a year, is the same as finding that term in a geometrical series, the same in number as the periods, whose first term is the principal, and common ratio the number representing £1, with its interest for the first period.

Similarly, to find the present worth of any amount due at the end of a given time, we must divide the amount

by 1, with its interest for a period, as often as there are periods.

Ex. 1. Find the amount of £10,000, at compound interest, for 4 years, at 3 per cent., payable yearly.

Here, $1 + \frac{3}{100} = \frac{103}{100}$ is 1, with its interest for a year, therefore, the amount will be $10000 \times \frac{103^4}{100^4} = \frac{103^4}{10,000} = \frac{112,550,881}{10,000} =$ £11,255 1 9.

Ex. 2. Find the present worth of £630 due two years hence, allowing 5 per cent. per annum, compound interest.

Here, $1 + \frac{1}{20} = \frac{21}{20}$ is 1, with its interest.

Therefore the pres. worth is $\frac{630}{\frac{21^2}{20^2}} = 400 \times \frac{10}{7} =$ £571 8 $6\frac{6}{7}$.

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## ANNUITIES.

96. An Annuity is a yearly amount to be paid once a year, or at stated times, and often for life.

Rule 1. To find the present worth of an annuity to continue for any number of years, allowing compound interest. Find the present worth of each payment, add all these sums together, this will give the present worth of the annuity.

Rule 2. To find the annuity a given amount will purchase. Find the present worth of £1 payable at the end of every period in the time the annuity is to continue. Divide the given amount by the result, this will give the annuity, or that part of it which will be received each period.

When the annuity is for life, the number of years, or the value of the life, must first be ascertained.

Calculations like these will be facilitated by the use of

Tables of Logarithms. Thus, in Ex. 1, Art. 95, we have the logarithm of the amount equal to the logarithm of $\frac{103^4}{10000}$ = log. $103^4$ − log. 10,000 = 4 log. 103 − 4, since the logarithm of 10,000 is 4.

Now, the register for 103 is ·01284, and as 103 has 3 figures, the complete logarithm will be 2·01284; but 4 × 2·01284 − 4 is 4·05136; and we find the number standing opposite ·05125 is 1125; making proper connections, and observing the characteristic is 4, and therefore there must be 5 figures in the whole number, we obtain £11,255 1*s.* 9*d.*

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## QUESTIONS.

A 1. What is meant by Arithmetical Progression? 2. How is the common difference found? 3. How any term? 4. How the sum of the series? 5. How may means be inserted between two terms?

B 1. What is Geometrical Progression? 2. How is the common ratio found? 3. How any term? 4. How the sum of the series?

C 1. What is the amount of £1, in one year, at 5 per cent.? 2. In two years? 3. In any number of years? 4. What is the amount of £5, at 5 per cent., in any number of years? 5. Of any number of pounds? 6. What would the multiplier be if the rate were 3 per cent., payable yearly? 7. What is the general conclusion? 8. How may present worth, at compound interest, be calculated? 9. What is an annuity? 10. Shew how to find its present worth. 11. Shew how to calculate how much must be paid for an annuity.

# CHAPTER X.

## MENSURATION AND DUODECIMALS.

97. In Mensuration we have to find the lengths of lines, and the contents of areas and solid figures, or to reduce them to numerical calculations.

This is accomplished by agreeing to call, firstly, a certain length the unit, or one, in the calculation of lines or lineal measure; secondly, a certain area the unit, or one, in the calculation of areas; and, thirdly, a certain volume the unit, or one, in the calculation of solid figures. Thus, if the unit of length be one inch and written 1, a line 36 inches long will be written 36, and, in like manner, areas and volumes are represented.

Nor can there be any confusion in thus writing numbers for such different quantities, as we can always tell when a number denotes length, or area, or volume; because we pass from one to the other according to laws and rules hereafter to be given, and, at every stage of our working, the numbers always represent the same kind of thing.

The units of area and volume are generally made dependent upon that of length, which is arbitrary.

The unit of length commonly taken is a line one inch long; the unit of area taken will be the area of a square side, one inch; and the unit of volume that of a cube side,

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97. The object of this short introduction being to recommend rather than prevent the study of pure Geometry, it is assumed that the student is acquainted with the names given to the different figures.

one inch ; and all other lengths, areas, and volumes whatever, are given as numbers and fractional parts of these.

98. **Rule.** To find the area of any rectangular figure. Multiply the number of units in one side by the number in one adjacent.

This product can be found by reducing feet to inches, and the parts to decimals, and then multiplying ; or by a rule called duodecimals.

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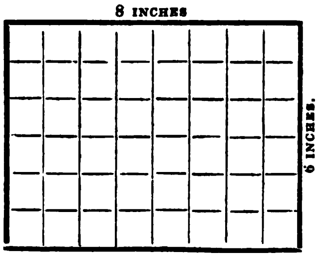
(98.) The reason of this will be seen from the following figure, where we have a rectangle whose adjacent sides are 8 inches and 6 inches respectively. Divide the sides into inches, and draw lines parallel through the points of division.




Then each of the figures will be a square side, one inch, or the unit of area (one inch being the unit of length), and as there are eight of these in one row, and in each of the rest, and there are six rows, therefore they will be 8 × 6 divisions in number.

This can be proved equally in any other case. Placing any rectangle with its lines parallel to the sides of the book, and dividing as before, we shall have as many rows as there are units in the line parallel the side of the book, and each of these rows will be divided into as many divisions as there are units in the line parallel the top of the book, therefore the number of divisions, that is, squares, of one inch each, will be the product of these numbers multiplied together.

It must be carefully borne in mind that it is not lines that are multiplied by lines, for to multiply lines by lines, or money by money, is absurd, but merely the number of units that are multiplied together, and the quotient represents units of an area, because we have made our unit such that it shall be so.

## DUODECIMALS.

99. **Rule.** Place feet under feet, inches under inches, &c., multiply by the highest denomination first, beginning at the lowest denomination of the multiplicand; consider the product to be in the lowest denomination, and bring it to a higher; write down the remainder, and carry the

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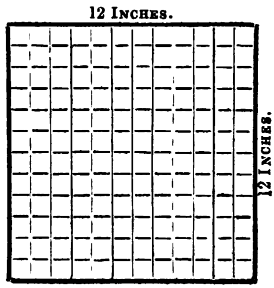
(99.) To understand this process taking (as the unit is arbitrary) a line one foot long for the unit, the figure representing a square side, one foot, will be the unit of area, and will contain twelve rows each containing twelve square inches: now each row has lines a foot long, and one inch long for its adjacent sides, and is the twelfth part of an area one foot square, that is, one foot by one foot, and just in the same way if it were three feet by two inches, it would be the twelfth part of an area three feet by two feet, and similarly for any numbers, therefore feet by inches being the twelfth part of feet by feet are put in the inches place, that is, in the place representing the twelfth part of a square foot. Similarly if we take an area one inch by one inch, or one square inch, we get the one hundred and forty-fourth part of a square foot; two inches by two inches, we get four square inches, or the one hundred and forty-fourth part of two feet by two feet, and so on; therefore inches by inches are put in the seconds place. In the same way, inches into seconds represent the $\frac{1}{1728}$ part of a square foot, and so on.




Hence, each place represents the area, contained by lines of the next higher denomination, and that of the place in which the number is.

In the Ex. representing the product in square feet it will be

$$36 + \frac{9}{12} + \frac{6}{144} + \frac{1}{1728} + \frac{6}{1728} \times 12, \text{ or } 36\tfrac{9}{12} \text{ sq. ft., } 6\tfrac{3}{24} \text{ sq. in.}$$

or 36 sq. ft. $87\frac{3}{24}$ sq. in.

quotient to add to the product of the next higher denomination and the first multiplier, and continue till all are multiplied.

Multiply by the next lower denomination, but place the first product one place nearer the right hand, and continue till all have been multipliers; considering there are 12 parts, or square seconds, in the inch, and these parts are again subdivided into 12 parts (or square thirds): add all the products.

| | feet. | in. | seconds. | | |
|---|---|---|---|---|---|
| | 7 | 2 | 6 | | |
| | 5 | 1 | 3 | | |
| | 36 | 0 | 6 | | |
| | | 7 | 2 | 6 | |
| | | 1 | 9 | 7 | 6 |
| ft. | 36 | 9 | 6 | 1 | 6 |

We may also multiply by the feet, and take parts for the inches and seconds by Practice.

It must be remarked that the number in the product standing in the first place represents square feet; but, in the place next lower, they are not square inches but twelfths of a square foot, and should be called square firsts; the number next lower under the seconds place, which we call square seconds, represents square inches.

100. **Rule.** To find an adjacent side of a rectangle, having given the area and one side. Divide the area by the given side.

Ex. A rectangle whose area is 14 ft., 10 sq. firsts, 6 sq. seconds, and one side 4 ft. 3 in., find the adjacent side.

| | ft. | sq. ft. | sq. s. | |
|---|---|---|---|---|
| 4 3) | 14 | 10 | 6 | (3 6 |
| | 12 | 9 | | |
| | 2 | 1 | 6 | |
| | 2 | 1 | 6 | |

Therefore the adjacent side is 3 ft. 6 in.

**101. To find the area of any parallelogram.** Let fall a perpendicular from one corner on the base (or base produced). Multiply the number of units in it by those in the base.

The reason of this is, that a parallelogram is equal to a rectangle, on the same base, and having the same height.

**102. To find the area of a triangle.** Find the length of the perpendicular from the vertex to the base. Multiply the number of units in it by the number of units in the base (or, in brief, multiply it by the base), and divide the product by two.

The reason of this is, a triangle is half a parallelogram, on the same base, and having the same altitude.

Ex. The base of a parallelogram is 2ft. 7in., height 1ft. 6in.

| ft. | in. | |
|---|---|---|
| 2 | 7 | |
| 1 | 6 | |
| 2 | 7 | |
| 1 | 3 | 6 |
| 3 | 10 | 6 |

A triangle right-angled, base 5ft. 8in., height 3ft. 4in.

| ft. | in. | |
|---|---|---|
| 5 | 8 | |
| 3 | 4 | |
| 17 | 0 | |
| 1 | 10 | 8 |
| 18 | 10 | 8 |

**103. To find the area of a rectangular solid.** Multiply the number of units in two adjacent sides of the base together

103. For conceiving the solid divided into rectangular cubes, with sides, one inch, each of these would be the unit of volume. Now the base would be divided as figure one, and the number of units of volume, if the solid were one inch high, would be the number of units of area in that case. Conceive the entire solid thus divided into rectangular slices one inch high; the number would be the number of units of length in the side perpendicular to the base. Therefore, we must multiply the number of units in the area of the base by the number of units in the side perpendicular; or, what is the same thing, find the product of the three sides which meet in one of the angles.

(that is, find the area of the base), and multiply this product by the side perpendicular to the base.

Ex. A rectangular solid 3ft. 6in. by 4ft. 3in. and 2ft. 4in.

| ft. | in. | |
|---|---|---|
| 3 | 6 | |
| 4 | 3 | |
| 14 | 0 | |
| | 10 | 6 |
| 14 | 10 | 6 |
| 2 | 4 | |
| 29 | 9 | 0 |
| 9 | 3 | 6 |
| 39 | 0 | 6 |

The contents may also be found by Decimals, or Practice.

Here the number under the inches represents twelfths of cubic feet, that in the seconds place $\frac{1}{144}$ of at cubic foot, and the next would be $\frac{1}{1728}$ of a cubic foot.

When the solid is not rectangular, but has its opposite faces parallel, find the area of the base (that is of one face), and multiply by the height.

104. To find the circumference of a circle. When the diameter is given, multiply it by 3·14159, or, as an approximation, $\frac{22}{7}$; or the radius by $\frac{44}{7}$.

To find the area of a circle. When the radius or diameter is given. Find the area of a square whose sides are equal to the radius, and multiply it by 3·14159 or $\frac{22}{7}$.

When the area is given to find the radius. Divide by 3·14159 or $\frac{22}{7}$, and then take the square root.

To find the area of an ellipse, or oval. Take the product of the semi-axes, and multiply by 3·14159.

105. To find the contents of cylinder. Find the area of the base, multiply it by the number of units in the height of the cylinder.

To find the contents of a cone. This is the $\frac{1}{3}$ of the

The ratio which the circumference of a circle bears to the diameter is 3,14159 nearly, and since $\frac{22}{7} = 3·142...$ for ordinary purposes $\frac{22}{7}$ is sufficient.

last volume; but to find it independently, find the area of the base, multiply by the number of units in the height, and divide the product by 3.

To find the volume of a sphere. Take the cube of the radius, and multiply by $\frac{4}{3}$ of 3·14159, that is, 4·18876.

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## GENERAL QUESTIONS.

A 1. Explain how Multiplication is in reality Addition. 2. How Division, Subtraction. 3. Why must the multiplier and divisor be abstract numbers? 4. How are quantities represented by numbers? 5. If the unit be 2*d.*, how will 6*d.* be written? 6. How if $\frac{3}{4}$*d.*?

B 1. Explain the nature of a fraction. 2. Why must fractions be reduced to a common denominator before adding or subtracting? 3. Why is a decimal called a decimal fraction? 4. Why may whole numbers and decimals be written with merely a point between? 5. What is the rule in Multiplication? 6. What in Division of Decimals?

C 1. What are the two general rules in Practice? 2. What is a ratio? 3. Why must the terms of a ratio be of the same kind? 4. What is Proportion? 5. How may *inverse* be written as *direct* proportionals? 6. Explain when two quantities and their Price are direct proportionals. 7. When inverse. 8. How are Principal and Interest proportionals.

D 1. What is the square root of a number? 2. What the cube root? 3. What is meant by a number raised to any power? 4. What by a root of a number? 5. What is a logarithm? 6. Explain the tables, and their use. 7. How may Compound Interest be expressed as a term of a geometrical progression?

E. 1. What is Mensuration? 2. How are lines, areas, and volumes represented by units? 3. Explain the rule called Duodecimals. 4. Which are square inches in the product? 5. How is the area of a rectangle found? 6. How of a cube? 7. Which are cubic inches in the product? 8. How must the area of a circle be found?

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